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Updating the Reliability of Concrete Structures subjected to Carbonation

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ABSTRACT

Due to the existence of uncertainties associated with mechanical properties, geometric configuration, loadings, and imperfect knowledge associated with the evaluations and predictions of material deterioration, probabilistic methods have been applied in the design and performance assessment of concrete structures to quantify these unavoidable uncertainties. This paper presents an approach for improving the accuracy in the life-cycle reliability assessment of reinforced concrete (RC) structures subjected to the carbonation by the Sequential Monte Carlo Simulation (SMCS). Using SMCS, multiple random variables related to observation information can be updated simultaneously, even if non-Gaussian random variables are involved and relationships between the observation information and random variables are nonlinear. The effect of the magnitude of inspected carbonation depth on the updated estimates of reliability associated with the occurrence of steel corrosion is discussed in this paper.

Keywords. Updating, Failure Probability, Sequential Monte Carlo Simulation, Carbonation, Reinforced Concrete

INTRODUCTION

As carbonation progresses, the corrosion could become serious enough to deteriorate not only the serviceability, but also the maintainability of the structural performance. When carbonation reaches at the depth of the rebar embedded into the concrete, the high alkalinity of the concrete pore solution is neutralised and hydration products are dissolved then to lower the buffering capacity of hydrations against a pH fall. At this moment, the passivation layer on the steel surface, which otherwise would protect the steel embedment from a corrosive environment, is destroyed, and steel is directly exposed to oxygen and water, eventually to corrode (Ann et al., 2010). Corrosion initiation could lead to cracking due to corrosion products and concrete cover spalling. Cracking and/or spalling accelerate the corrosion rate and finally lead to serviceability failure and a deterioration of long-term structural performance. Such deterioration will reduce the service life of structures and increase the life-cycle cost of maintenance actions. Various environmental and mechanical stressors, and structural variables affect the degradation mechanism of RC structures subjected to the carbonation.

However, the effects of stressors and variables are very difficult to predict, as these effects vary in time and space. Because of the presence of uncertainties, it is necessary that structural long-term performance be treated based on reliability concepts and methods (Ellingwood, 2005 and Frangopol, 2010). Stochastic treatment of structural design problems takes into account the uncertain nature of structural performance making a reliable design of RC structures possible (Frangopol et al.,1997 and RILEM,1998).

Life-cycle reliability estimation of RC structures depends on many aleatory and epistemic uncertainties involved in the evaluation and prediction of carbonation progress. For existing structures it is possible to reduce epistemic uncertainties using visual inspection, nondestructive inspection, and/or monitoring results, and to improve the estimation accuracy of the present and future failure probability. However, the inspection and/or monitoring results relate to many random variables. When relationships between the inspection and/or monitoring results and random variables are nonlinear or non-Gaussian variables are involved in the life-cycle reliability estimation, a rigorous theoretical approach is generally impossible to implement in realistic cases. An approximate solution can, however, be found by using several approaches. The Monte Carlo approach is in general used because of its versatility. MC based methods for non-linear filtering techniques have been developed since the 1990s. These methods include MC filter, bootstrap filter, recursive MCS, sequential MCS, the sampling importance resampling method, and the sequential importance sampling with resampling (Gordon et al., 1993, Kitagawa, 1996 and Ristic et al., 2004).

In this paper, life-cycle reliability analysis of RC structures subjected to the carbonation is conducted. Failure probability is estimated by the limit state comparing the carbonation depth with concrete cover. To improve the accuracy in the life-cycle reliability assessment of RC structures, multiple random variables related to the inspected carbonation depth are updated using the Sequential Monte Carlo Simulation (SMCS). The effect of the magnitude of inspected carbonation depth on the updated estimates of reliability associated with the occurrence of steel corrosion is discussed in the illustrative example.

SEQUENTIAL MONTE CARLO SIMULATION

Even though aleatory uncertainty cannot be reduced, improvement in our knowledge or in the accuracy of predictive models will reduce the epistemic uncertainty (Ang and De Leon, 2005). This means that for existing structures, the uncertainties associated with predictions can be reduced by the effective use of information obtained from visual inspections, field test data regarding structural performance, and/or monitoring. This information helps engineers to improve accuracy of structural condition prediction. However, the updated random variables do not follow, in general, widely used PDFs (such as normal, lognormal, etc.). The difficulties of the solution in Bayesian updating depend on the relationships between observed physical quantities, such as inspection results, and the PDFs of associated random variables. In this study, SMCS is applied to the updating of random variables. The reliability estimation using SMCS is briefly described as follows. The detailed procedure was given by Yoshida (2009).

The state space model consists of two processes, the time updating process and the observation updating process. The time updating process is the one step ahead prediction based on the information at the (k-1)-th step. The predicted state vector is

$$x_{k/k-1} = F(x_{k-1/k-1}, w_k) \tag{1}$$

where w_k is system noise represented by the noise involved in the prediction process. It is assumed that observation information z_k is a function H of state vector $x_{k/k}$ and observation noise v_k as

$$z_k = H(x_{k/k}, v_k) \tag{2}$$

The PDF of these noises, $p(w_k)$ and $p(v_k)$, are assumed known and independent. The real world problems, however, often involve nonlinearity and non-Gaussian noises. Figure 1 shows the flowchart of the method. The flowchart starts by assuming samples drawn from the distribution at (k-1) th step

$$x_{k-1/k-1}{}^{(j)} \approx p\left(x_{k-1/k-1} \middle| Z_{k-1}\right), \quad j = 1, \cdots, n \tag{3}$$

$$Z_{k-1} = (z_1, z_2, z_3, \cdots, z_{k-1}) \tag{4}$$

The superscript (j) denotes the generated j-th sample realization. The PDF is approximately expressed by the samples with Dirac delta function δ as

$$p(x_{k-1} | Z_{k-1}) \cong \frac{1}{n} \sum_{j=1}^{n} \delta(x_{k-1} - x_{k-1/k-1}{}^{(j)}) \tag{5}$$

The above approximation form of PDF is called empirical PDF. The samples of the k-th step before observation updating are obtained by simply substituting them into state Equation (1)

$$x_{k/k-1}{}^{(j)} = F(x_{k-1/k-1}{}^{(j)}, w_k{}^{(j)}) \tag{6}$$

The empirical PDF of the k-th step before updating is similarly estimated by the sample realization

$$p(x_{k-1} | Z_{k-1}) \cong \frac{1}{n} \sum_{j=1}^{n} \delta(x_{k-1} - x_{k-1/k-1}{}^{(j)}) \tag{7}$$

The PDF after updating is

$$p(x_k | Z_k) = p(x_k | z_k, Z_{k-1}) = \frac{p(x_k, z_k | Z_{k-1})}{p(z_k | Z_{k-1})} = \frac{p(z_k | x_k, Z_{k-1}) \cdot p(x_k | Z_{k-1})}{\int p(z_k | x_k, Z_{k-1}) \cdot p(x_k | Z_{k-1}) \cdot dx_k} \tag{8}$$

The term $a_k^{(j)}$ is the weight (likelihood ratio) of sample j. When a new observation is available, the weights are re-calculated and the approximate posterior PDF is sequentially updated. However after a few steps the confidence in the estimated PDF deteriorates. This is often called weight degeneracy or sample impoverishment. To alleviate this problem, a resampling step is introduced by Arulampalam et al. (2002) and Ristic et al. (2004).

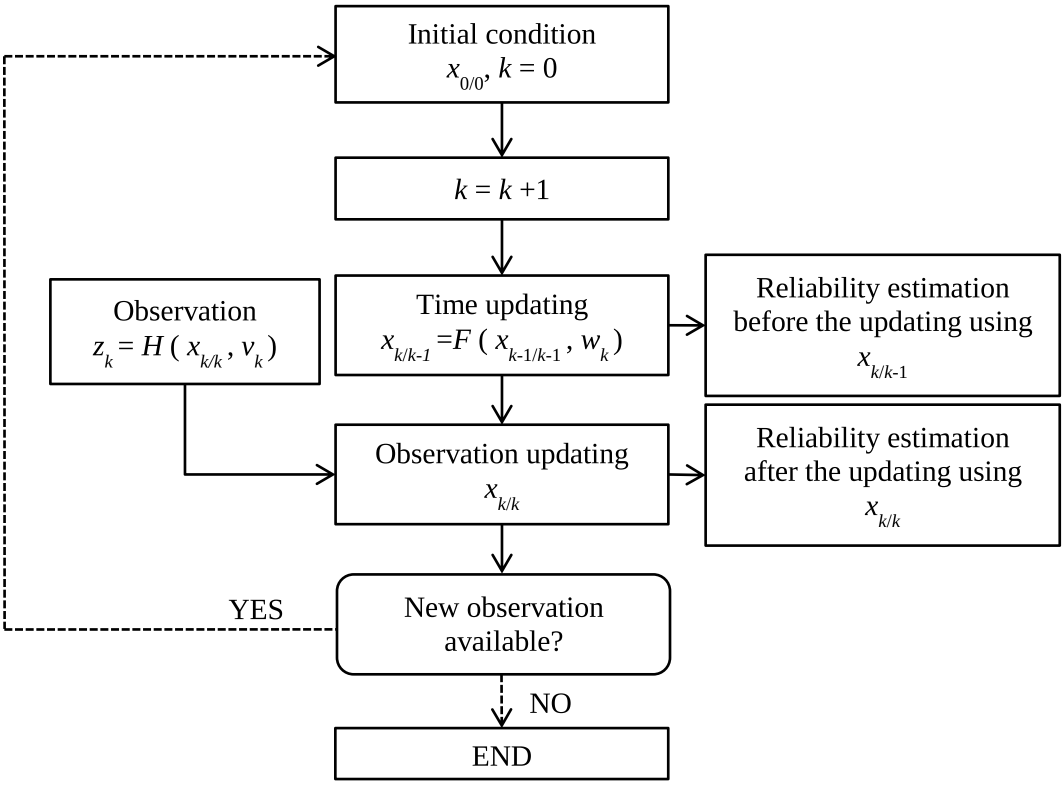


Figure 1. Flowchart of reliability estimation using SMCS

ILLUSTRATIVE EXAMPLE

Deterioration of Concrete. Many researchers suggested that the carbonation of RC structures could be divided into four stages (Sung et al., 2010). In the first stage, the initiation stage, the carbonation depth of concrete has not yet reached a critical threshold. The rebars do not begin to corrode, so that the degradation of structural performance is not significant. As carbonation propagates, the corroded rebars in the second stage, the propagation stage, tend to grow in volume and generate a dilative pressure towards the surrounding concrete. At the same time, the effective cross-sectional area of the corroded rebars has been reduced. At the end of the propagation stage, the corrosion amount of rebar reaches a threshold value, resulting in the cracking of the cover concrete. If carbonation continues, the structure will further deteriorate due to the widening cracks, as depicted by the acceleration stage. Finally, severe degradation of structural performance will occur in the deterioration stage due to intensive cracks in the concrete and severe corrosion of the rebars. For each carbonation stage, if concrete properties, corrosion depths of rebars and the varied strength of rebars can be determined quantitatively, time-dependent structural capacity can be calculated. However, it is very difficult to evaluate the steel weight loss and its spatial distribution in concrete structures subjected to the carbonation. In this study, reliability analysis for concrete structures in the initiation stage is conducted. Failure probability is estimated using the performance function comparing the carbonation depth with concrete cover. That is

$$\begin{aligned} g &= R - S \\ &= x_1 - x_2 \cdot X \end{aligned} \tag{9}$$

$$X = \alpha \cdot \kappa \cdot \gamma \cdot \sqrt{t} \tag{10}$$

$$\gamma = (x_3 \cdot WC - 0.25) / \sqrt{0.3(1.15 + 3 \cdot x_3 \cdot WC)} \tag{11}$$

where x_1 is the construction error associated with concrete cover, x_2 is the random variable representing model uncertainly. X is the depth of carbonation (Izumi, 1988, Nakayama and Matsubara, 1992, and Chien et al., 2008), α is the environment coefficient, κ is the material coefficient, γ is the coefficient associated with water to cement ratio, x_3 is the construction error associated with water to cement ratio, and WC is water to cement ratio. In the illustrative example, α and κ are assumed to be 1.0. Table 1 shows assumed statistics and probabilistic distribution of random variables x_1 to x_3.

Table 1. Parameters of random variables

Variables		Distribution	Mean	COV	STD
x_1	Water to cement ratio	Normal	0.45	-	0.1
x_2	Estimated value of carbonation depth	Lognormal	*	0.4	-
x_3	Concrete cover	Normal	30mm	-	10mm

Table 2. List of inspection date

Case	Observation quantity	Year of inspection	Inspected carbonation depth
0	-	-	-
1	Depth of carbonation	30 years	1.02mm
2	Depth of carbonation	30 years	5.91mm
3	Depth of carbonation	30 years	16.1mm

Modeling of Observational Date. It is assumed that the carbonation depths are provided by the phenolphthalein test at the 30 years after construction. In the illustrative example, Cases 1 to 3 with different carbonation depths are considered as listed in Table 2. Carbonation depths of Cases 1, 2 and 3 are 5%, 50%, and 95% percentiles of those estimated by ordinary MCS with random variables listed in Table 1, respectively. SMCS is not used in Case 0. From Equations (10) and (11), when depth of carbonation by the phenolphthalein test is given, the observation equation based on the observation date (i.e., carbonation depth) is

$$z = x_2 \cdot \alpha \cdot \kappa \cdot \frac{x_3 \cdot WC - 0.25}{\sqrt{0.3(1.15 + 3 \cdot x_3 \cdot WC)}} \cdot \sqrt{t} + v \tag{12}$$

where z is the observed depth of carbonation from the concrete surface, and v is the observation noise. v is assumed to be a standard normal distribution with the standard deviation of 1.0.

Time-Dependent Reliability Analysis of RC Structure. Computational results given by SMCS are almost the same for all cases if the number of samples is more than 500,000. Therefore, the number of samples is set to 500,000. Figure 2 shows the relationship between the failure probability associated with the occurrence of rebar corrosion due to the carbonation and time after construction. In addition, the updated failure probabilities using the inspected carbonation depth at 30 years are shown in Figure 2. Since depth of carbonation provided by the phenolphthalein test in Cases 1 and 2 are less than that expected by random variables listed in Table 1, the failure probability at 30 years after updating is much smaller than that before updating (Case 0). However, the failure probability in Case 3 is much higher than that in Case 0.

Correlations between *R* and *S* used in Equation (9) for Case 2 are shown in Figure 3. Figures 3(a) and (b) show *R* and *S* before and after updating at 30 years, respectively. All parameters of random variables related to the observational data can be updated by SMCS simultaneously using the joint probability density functions(PDFs) of the random variables (including means, COVs and correlations). *R* and *S* are statistically independent before updating. As these two random variables are updated using the same inspection results, it is confirmed in Figure 3 that they have to be correlated after updating.

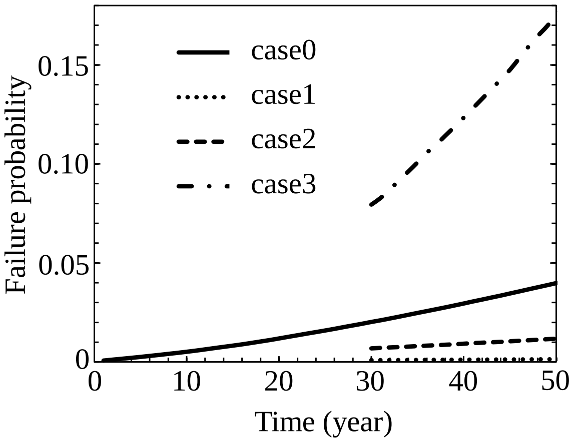
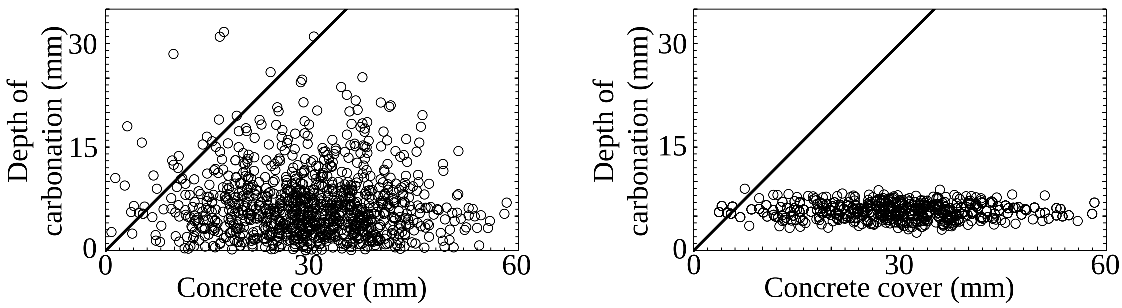


Figure 2. Relationship between time and failure probability



(a) before updating **(b) after updating**

Figure 3. Relationships between *R* and *S* before and after updating at 30 years (Case 2)

CONCLUSIONS

The procedure to obtain the failure probabilities of concrete structures subjected to carbonation using SMCS was presented in this study. The carbonation depth provided by the phenolphthalein test was used as observational data. The effect of the magnitude of inspected carbonation depth on the updated estimates of reliability associated with the occurrence of rebar corrosion was discussed in the illustrative example.

Using SMCS, multiple random variables related to observational information can be updated simultaneously. Updating makes possible to improve the accuracy in the life-cycle reliability assessment of concrete structures. Further research is needed on the reliability of RC structures subjected to both the carbonation and other environmental stressors such as airborne chloride. Also, spatial distribution of carbonation over the entire concrete structures have to be considered.

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