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Regression Function to Predict Tension Softening Curve for Concrete of Arbitrary Mix Proportion

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ABSTRACT

A reliable tension softening curve is the essential constitutive law for the purpose of crack analysis of concrete in order to realize sustainable infrastructures. In this paper, such polynomials are proposed to predict the tension softening curve for concrete of any arbitrary mixture based on the reliable test data obtained by the authors. Three variables of mixture, i.e. W/C, s/a and G_{max} were selected which are considered to affect the tensile properties of concrete most. The mixture of usual concrete is within a rectangular prism in three-dimensional space of the three variables. At first the interpolation function of 8 term polynomials was adopted to predict an inside value of the rectangular prism by 8 data of the apices. After additional regression analyses, the linear polynomials of 4 terms were finally selected as the best regression function for the prediction in order to avoid excessive influences by the experimental error.

Keywords. Tension softening curve, Uniaxial tension, Concrete, Prediction, Regression

INTRODUCTIONS

In the analysis of shrinkage cracks which is essential in durable concrete and sustainable constructions, the tension softening curve is indispensable. The best way to obtain the tension softening curve is to perform a uniaxial tension test. This test directly provides the tension softening curve and tensile strength from an identical specimen without any inverse analysis. However, there are only few investigators to perform the test. Because, the test needs an expensive loading machine and special equipments to minimize some inevitable flexures that occur during the test.

The authors have proposed a database for the prediction of the tension softening curve of an arbitrary concrete made from crushed andesite based on the experimental data (Akita, 2010). With such a database, anyone can predict the tension softening curve without performing the test only when the mixture of the concrete is known. Such database is very beneficial for researchers and designers who want to analyze crack behavior or fracture behavior of some concrete structures.

In order to obtain a good prediction from the database, some regressions, but not interpolations, should be adopted and based on some reliable test data. Because, actual data always include some experimental error whereas an interpolation is adequate only when the experimental data are exact. The authors have established a reliable test method within the last decade to provide the necessary data for the database (Akita, 2003, 2005). Three representative variables of mix proportion, i.e. W/C, s/a and G_{max} were selected which are considered to affect the tensile properties of concrete most. The mix proportion of usual concrete is within a rectangular prism in three-dimensional space of the three variables.

In this paper, the process to arrive to the best regression function is presented by using some concrete examples. At first the interpolation function of 8 term polynomials was adopted to predict an inside value of the prism by 8 data of the apices. Then 7 reference points were added in the center of the 6 faces and the center of the prism and the constants of the regression function were determined by the least square method. Also the perfect quadratic polynomials of 10 terms were tried to reduce the residuals of the experimental data. Finally the perfect linear polynomials of 4 terms were selected as the best regression function for the prediction in order to avoid excessive influences by the experimental error.

EXPRESSION FOR TENSION SOFTENING CURVE

In order to determine a basic function to express the tension softening curve, 21 tension softening curves shown in Figure 1 were adopted. They were obtained from the experiments using the specimens of an identical mixture during 2001 to 2005. The curves are expressed by a normalized form of σ_N and w_N in order to make it easy to get the average of the curves. Where σ_N equals σ / f_t, σ is tensile stress or cohesive stress and f_t is tensile strength, and w_N equals w/w_c, w is crack opening displacement (COD) and w_c is critical crack opening displacement. Some basic functions were proposed by Reinhardt et al. (1986) and Li et al. (2002). However, both of the proposed functions do not match the experimental data of the authors directly. Thus, the following basic function (Equation 1) was assumed by improving the function proposed by Li et al.

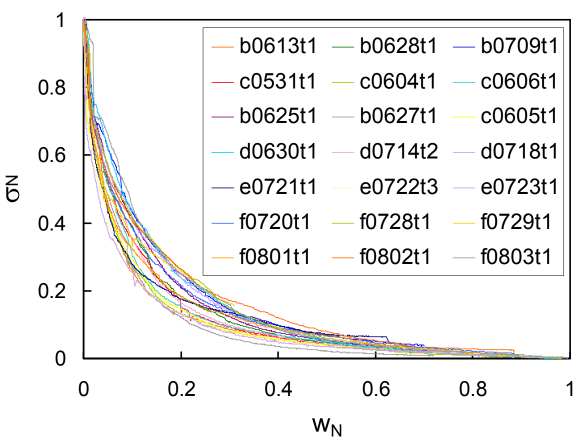

Figure 1. 21 tension softening curves

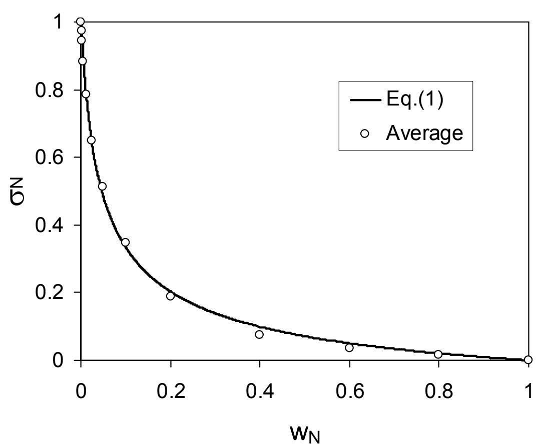

Figure 2. Approximation curve by least square

$$\sigma_N = -EXP\left(-\left(\frac{B}{w_N}\right)^A\right) + \frac{C}{w_N + D} + E \tag{1}$$

where A to E are constants or undetermined coefficients. After the 21 curves were averaged in 11 points (open circles) shown in Figure 2, these constants were determined by applying the least square method to fit the curve to the points.

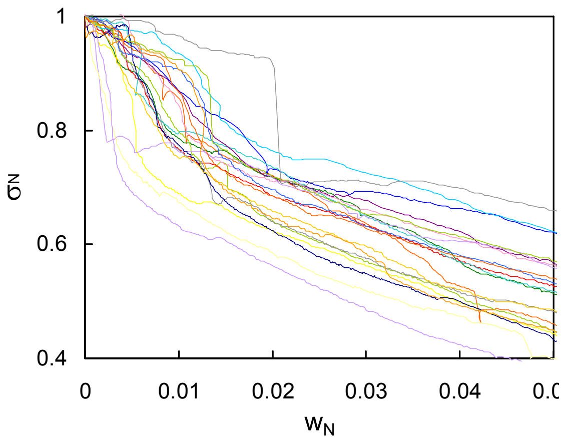

Figure 3. Initial parts of 21 curves

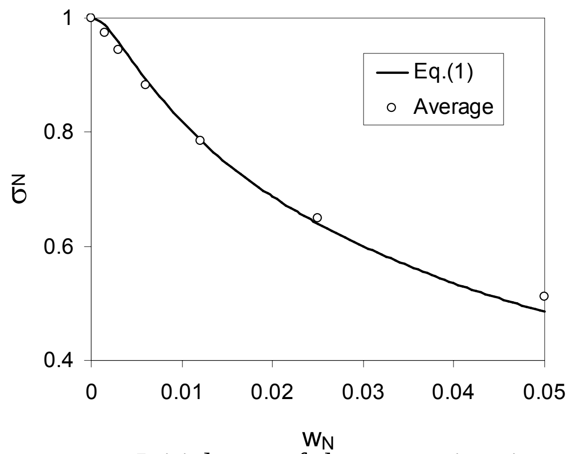

Figure 4. Initial part of the approximation curve

The resulting curve agrees with the points precisely and is shown in Figure 2. In Figure 3 and 4, the initial parts of the curves in Figure 1 and 2 are multiplied and shown, respectively. Because of the convex shape of the initial part of these curves, the first term in Equation 1 was adopted. In addition, it is supposed that Equation 1 can be applied to any concrete made from crushed andesite when the constants A to E are appropriately chosen.

REFERENCE MIXTURES

The reference mixtures of concrete to use for the prediction of tension softening curves of any arbitrary mixture were determined as follows. Three variables of mixture such as water cement ratio W/C, sand aggregates ratio s/a and the maximum size of coarse aggregate G_{max} were selected, because they were considered to affect the tensile properties of concrete most. As an ordinary concrete is within the range of W/C=40% to 60%, s/a=35% to 45% and G_{max}=15mm to 25mm, the range is shown like the inside of the rectangular prism in Figure 5 by 3-dimensional expression of 3 variables. Experiments were performed for 8 mixtures correlating to the apices of the rectangular prism at first, and then 6 mixtures were added correlating to the centers of all faces and the one other mixture correlating to the center of the prism. The prediction is the same procedure to determine the unknown value relating to an arbitrary point based on the known values of the reference points. The 15 reference mixtures for the present experiment are shown in Table 1 and Figure 5. Crushed andesite was adopted as coarse aggregates, because it was the most common in Japan.

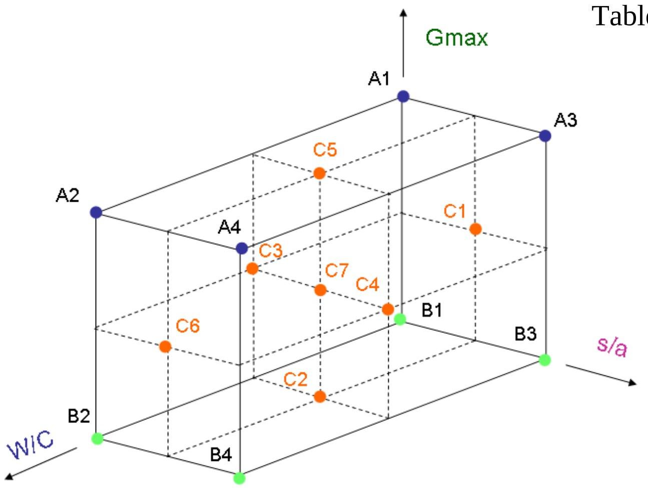

Figure 5. 3-D expression of 15 mixtures

Table 1. Mixtures of the reference concrete

Mix	W/C (%)	s/a (%)	G_{max} (mm)	W (kg/m^3)
A1	40	35	25	142
A2	60	35	25	143
A3	40	45	25	150
A4	60	45	25	155
B1	40	35	15	160
B2	60	35	15	146
B3	40	45	15	160
B4	60	45	15	160
C1	40	40	20	160
C2	50	40	15	158
C3	50	35	20	146
C4	50	45	20	152
C5	50	40	25	153
C6	60	40	20	157
C7	50	40	20	163

EXPERIMENT

Five cylinders of ϕ100x200mm for the compression test, the same 5 cylinders for the splitting tension test and 5 prisms of 100x100x400mm for uniaxial tension test were cast from one mixture. The compression test and splitting tension test were performed at the age of 28 days following after the Japan Industry Standard. The

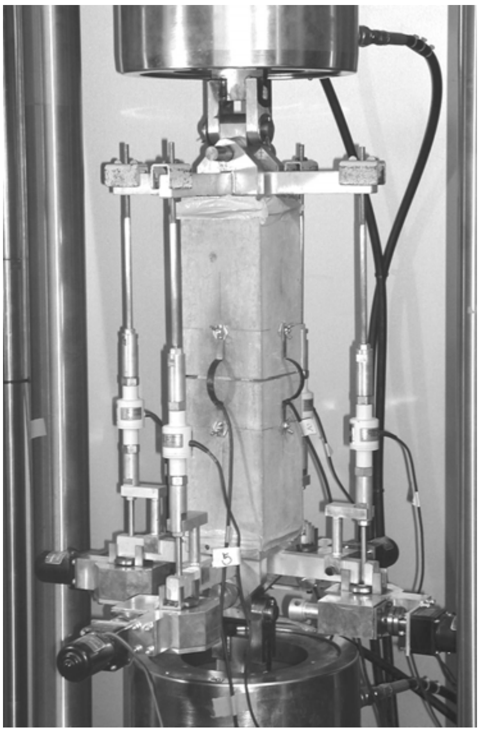

Figure 6. Experimental set-up

uniaxial tension test was performed using a strain-controlled loading machine as shown in Figure 6. Flexures caused by load eccentricity and heterogeneity of concrete were both minimized by adjusting gear systems. In Figure 6, the adjusting gear systems and extensometers on four side faces are shown. The minimization of flexures is executed as follows during the test. When a certain side of a specimen is elongated more than the opposite side, the more elongated side should be contracted by turns of its adjusting gear until reaching a proper balance in elongation. Such a real time execution is indispensable to minimize flexures caused by the heterogeneity of concrete. Three hours per specimen were spent including necessary preparation. Thus, the tension tests of 5 specimens were performed on the age of 29 and 30 days.

TEST RESULTS

An example of load-deformation (P-δ) curves directly obtained from the uniaxial tension test belonging to a specimen of mixture B3 are shown in Figure 7. Ch2 and ch4 in the Figure mean two opposite face deformations and the nearly exact overlap of the two curves indicates that flexures are completely minimized. Figure 8 shows an example of 5 tension softening curves derived from the load-deformation curves of mixture B3.

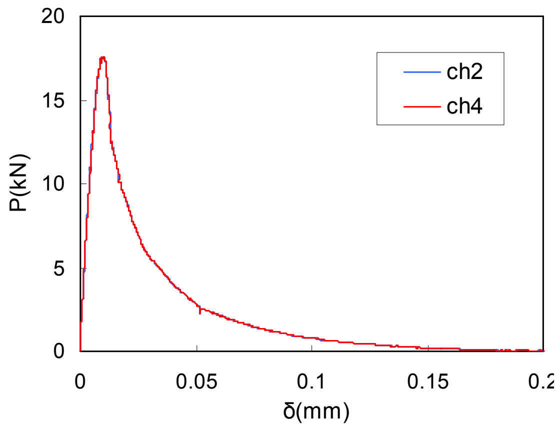

Figure 7. Load-deformation curves (h1207t1)

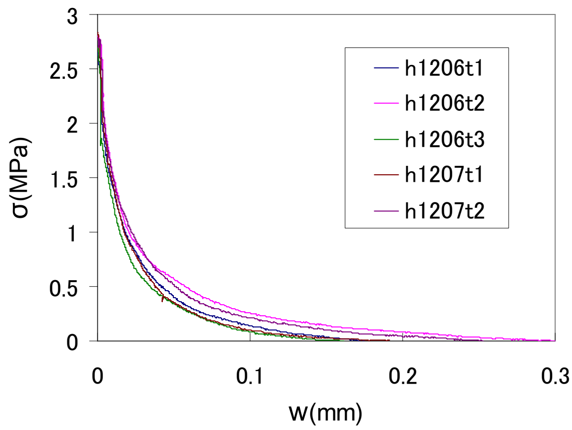

Figure 8. Tension softening curves (Mix B3)

PREDICTION BY INTERPOLATION FUNCTION

In order to predict the constants A to E in Equation (1) and the fracture mechanics parameters f_t, w_c and G_F at any arbitrary point inside of the prism in Figure 5, Equation 2 was adopted at first in which V represents any constants and fracture mechanics parameters.

$$V(x,y,z) = a + bx + cy + dz + exy + gyz + gzx + hxyz \qquad (2)$$

where a to h are constants or undetermined coefficients and x, y and z mean arbitrary values of W/C, s/a and G_{max} respectively. Equation 2 is equivalent to a linear Lagrange interpolation formula modified to a 3-dimensional form. The constants a to h are determined by simultaneous 8 equations referring to x ,y, z and V of 8 apices. Then, V of arbitrary x, y and z is calculated by Eq.(2), i.e. V is predicted from 8 values of V on 8 apices.

Figure 9 is an example concerning to f_t, where symbols indicate experimental values or reference values and lines are predicting values along the correlated edge of the prism. In rough observation, 4 lines are close to each other and intersect all the symbols, suggesting that the interpolation seems to give a good prediction. However, the experimental error is blindly ignored when the number of reference points or number of data equals to the number of undetermined coefficients in interpolation function. In other words, it means that such interpolation regards erroneous experimental values as exact ones and results to an unreliable prediction informing nothing about the experimental error.

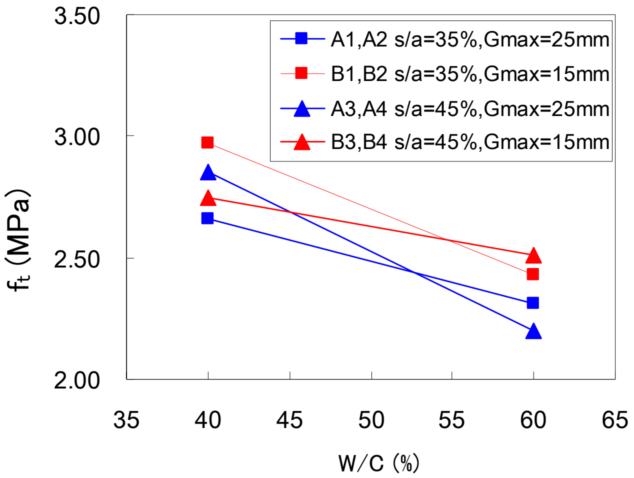

Figure 9. Prediction of f_t by Eq.(2)

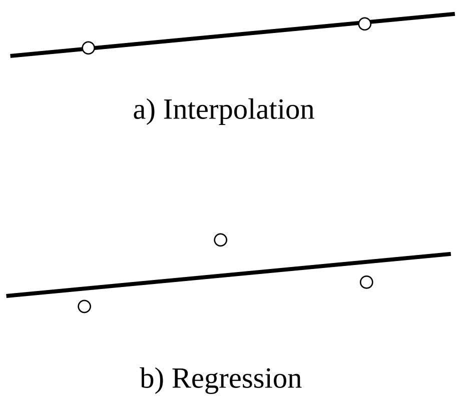

Figure 10. Prediction in one-dimensional case

It can be easily seen in simple one-dimensional examples in which Eq.(3) is a possible interpolation function and number of undetermined coefficients is two.

$$V(x) = a + bx \tag{3}$$

When there are two reference points or two experimental data, the interpolation line becomes like Figure 10 a), and everything seems to be perfect. However, the coincidence of the line and two points mean neither the prediction is exact nor the experimental data are exact. On the other hand, the regression line in Figure 10 b) is more reliable than the former when one more reference point is added. This is because the regression line is the line in which the total residual becomes minimum based on three data including experimental error, but the interpolation line simply connect the two points without considering experimental error.

PREDICTION BY REGRESSION FUNCTIONS

In order to improve the prediction reliability, 7 reference mixtures, C1 to C7, were added to the original 8 mixture, A1 to A4, B1 to B4, as shown in Figure 5. As the number of constants a to h in Eq.(2) is smaller than the number of the reference points, the constants are determined by the least square method referring to 15 values of the reference mixtures.

Figures 11 and 12 are the examples of the prediction concerning to fracture energy. In order to avoid complexity, they are shown by two Figures, namely the cases when s/a=35% in Figure 11 and when s/a=45% in Figure 12. By these two Figures, the variations of fracture energy in the prism concerning to 10 reference mixtures are expressed. In these Figures, the deviations of the experimental values (symbols) from the prediction values (lines) are small in apices (A1 to B4), but a little larger in the centers (C3 and C4). This suggests that the quadratic terms of x, y and z are necessary in the regression function.

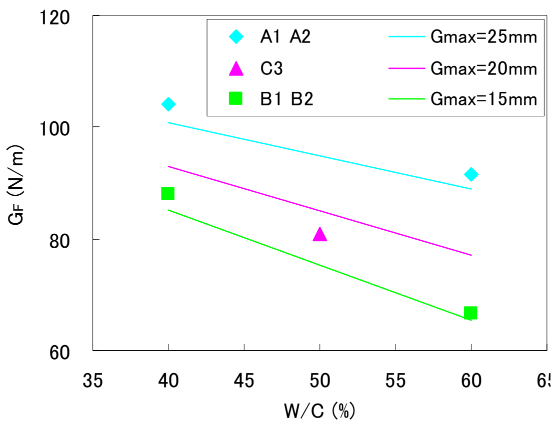

Figure 11. Prediction of G_F by Eq.(2) (s/a=35%)

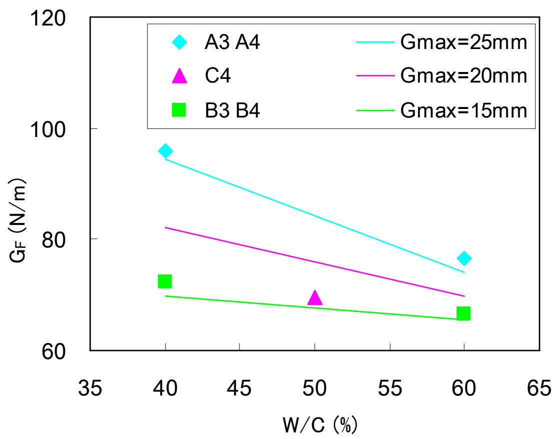

Figure 12. G_F by Eq.(2) (s/a=45%)

So, such terms were added and the perfect quadratic polynomials shown in Eq.(4) were adopted as a prediction function next.

$$V(x,y,z) = a + bx + cy + dz + exy + gyz + gzx + hx^2 + iy^2 + jz^2 \quad (4)$$

An example which is concerning to the same mixtures as Figure 12 is shown in Figure 13. The curved regression lines are well fitting to the experimental values. This figure suggests that an excellent prediction was obtained by Eq.(4). However, it is revealed by another example that this recognition is erroneous.

Figure 14 shows the variations of tensile strengths when s/a=35%. It also seems to be a very good prediction. However, the following consideration arrives at the opposite conclusion. It is well known that the compressive strength f_c of concrete is proportional to C/W namely 1/x. On the other hand, the previous experiments by the authors show the relationship between f_t and f_c as Figure 15 and Eq. (5).

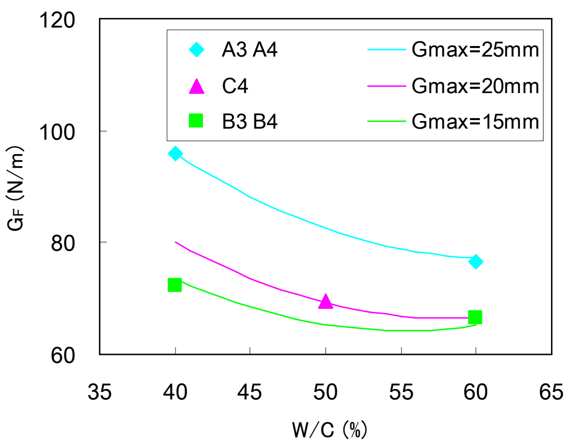

Figure 13. Prediction of G_F by Eq. (4) (s/a=45%)

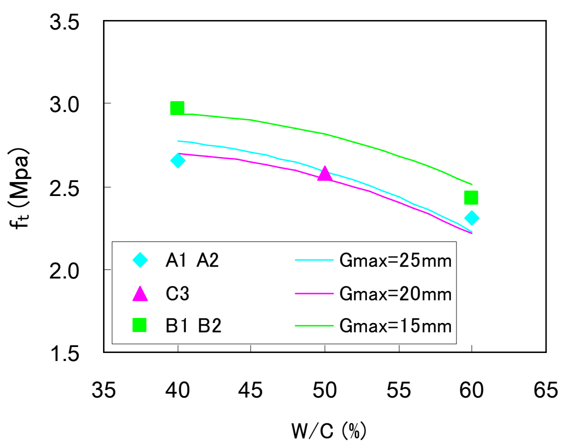

Figure 14. Prediction of f_t by Eq. (4) (s/a=35%)

$$f_t = 0.245 f_c^{0.8} \tag{5}$$

From the both relationship, f_t should be proportional to $x^{-0.8}$ and convex to the downside despite that Figure 14 shows convex curves to the upside. It means that the regression lines are excessively affected by experimental error and reach an erroneous prediction. In general, it does not mean a better prediction when small deviations between experimental values and prediction values are established by increasing high order terms in regression function. In fact, if high order terms are added until the number of undetermined coefficients becomes the same as the number of reference data, all the deviations are completely zero but the prediction is rather erroneous. The situation is the same as Figure 10 a) when it is one-dimensional problem.

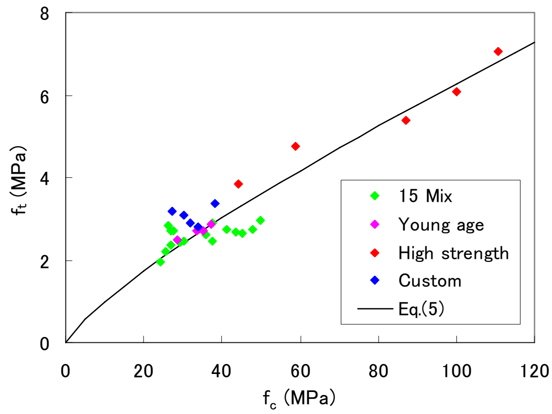

Figure 15. Relationship of f_t and f_c

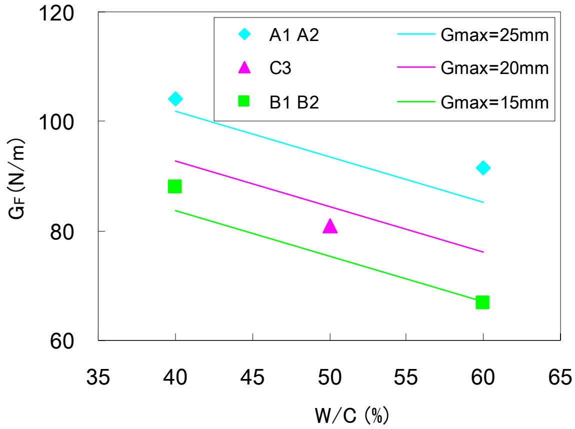

Figure 16. Prediction of G_F by Eq.(6) (s/a=35%)

Finally, one more regression function which is the perfect linear polynomials of 4 terms was adopted as shown in Eq. (6).

$$V(x,t,z) = a + bx + cy + dz \tag{6}$$

The examples by this regression function are shown in Figure 16 and 17. The deviations of the experimental values from the prediction values appear larger than the former examples. However it remains indecisive which is a better prediction function Eq. (2) or Eq. (6). Unfortunately, the decisive factors are not so strong.

One factor is that the variation in G_F by using Eq. (2) is not the same along the edges for example when comparing two s/a in Figures 11 and 12. Where the gradient is steeper in A3-A4 than in A1-A2 but it is less steep in B3-B4 than in B1-B2. The existence of these mutually opposite variations cannot be denied but it is considered unnatural. A possible consideration is that such opposite variations come from the reflection of the experimental error.

Next factor is that 15 reference points are not enough to decide the curvature or gradient variation in 3-dimensional space. It means that an excessive reflection of the experimental error loses the common variation among all the reference points. Thus, the final conclusion is that the most appropriate regression function in this study is Eq. (6)

PREDICTION RESULT

Figure 18 shows an example of the comparison of the predicted tension softening curve and the experimental curve. The predicted curve is calculated using all predicted values A to E, f_t and w_c, and the experimental one is the simple average of the five tension softening curves as shown in Figure 8. In this example the deviation of the two curves is the largest in 15 mixtures. In spite of being the largest, the deviation is within the scattering of five curves in one mixture as shown in Figure 8. This shows that the present prediction is excellent despite that individual deviation of the constants A to E, f_t and w_c between predicted and the experimental are prominent.

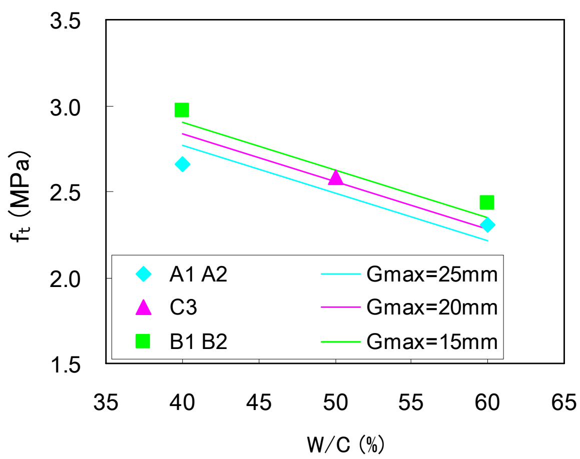

Figure 17. Prediction of f_t by Eq.(4) (s/a=35%)

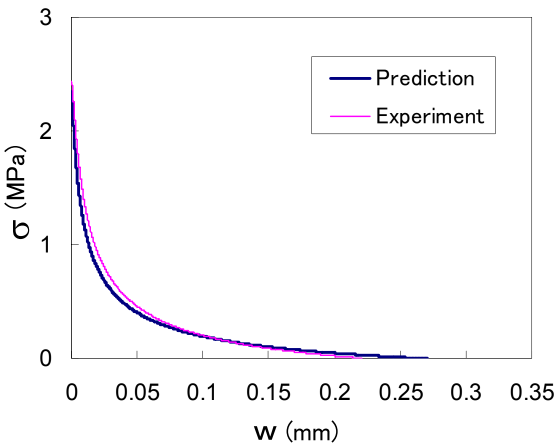

Figure 18. Comparison of two curves

CONCLUSIONS

A database to predict tension softening curve for concrete of arbitrary W/C, s/a and G_{max} without performing a tension test was proposed. In the prediction based on 15 experimental data, the regression function of the perfect linear polynomials of 4 terms is considered to be the best.

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