Third International Conference on Sustainable Construction Materials and Technologies http://www.claisse.info/Proceedings.htm

Theoretical considerations on estimation of compressive strength of concrete by means of hammer blowing

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ABSTRACT

There are two methods for estimating the compressive strength of the concrete by hammering the concrete surface. One is the rebound hammer and the other is newly developed mechanical impedance method. In this paper, the considerations on the theory of both two methods are described. Both methods are similar from the view point of hammering. However there is a great difference between both methods. The mechanical impedance method employs the direct drive method, whereas the rebound hammer employs indirect driving method by using a plunger. Therefore, the theories applied for analyzing those measuring methods are also different from each other. The theory of the rebound hammer should be based on the stress wave theory due to the plungers function and the motion equation will be applicable to analyze dynamic behaviours of the mechanical impedance method.

Keywords. Schmidt hammer, NDT, Strength of concrete, Stress wave Theory

INTRODUCTION

The rebound hammer method was developed almost 60year ago (K. Gaede, E. Schmidt,1960) and this method is used conventionally. However the accuracy of this method seems to be not established. At the first developing stage of this method, direct hammering method was employed to measure the rebound number as the rate between initial stroke of the mass and rebound stroke of it. As the method to estimate the compressive strength of the concrete, the idea of Brinell hardness was introduced, and then the classical theory of the rebound hammer method had been established. The Brinell hardness is the index value of the surface hardness and this method is mainly used for metal materials. Basically the Brinell hardness gives the coefficient of volume compressibility. However, we have some doubt on that the rebound hammer method gives the Brinell hardness. There are two problems regarding the theory of the rebound hammer method. The first is the question on the classical energy equilibrium theory whether it stands good or not. The other is the fact that the blowing mechanism of today's actual rebound hammer is different from the blowing mechanism in the theory of the rebound hammer.

With the original rebound hammer and newly developed mechanical impedance method, the hammer mass directly stroked the concrete surface, but nowadays rebound hammer has a plunger of the elastic bar, and the hammer strikes the head of the plunger. The hammer does not directly strike the concrete surface. That is, the driving force is determined with the kinematic condition of the hammer and the plunger and it is not affected by the strength of the concrete surface. This means that there are some differences between the actual blowing mechanism and the theoretical blow mechanism. Therefore it is needed to reform the theory of the rebound hammer and to re-analyse the applicability of the rebound hammer again to make clear what the rebound hammer measures.

In this paper, theoretical analysis of the classical rebound hammer method and the mechanical impedance method, the results of stress wave theory analysis of the dynamic behaviour of the rebound hammer and the fact that the rebound number indicates the elastic property of the concrete surface are shown.

CLASSICAL THEORY OF THE REBOUND HAMMER

Energy Equilibrium. Classical theory of the rebound hammer, the energy equilibrium when the hammer collides with the concrete surface, equation (1) is deduced.

$$E_0 = E_P + E_R \tag{1}$$

Where E_0, E_P, E_R are the initial kinematic energy of the hammer, the consumed energy by the plastic deformation of the concrete surface and the residual energy which is used for hammer rebounding. The hammer is driven by the spring with spring constant U. Let x_0 be the initial displacement of the spring and x_R be the displacement after when the hammer is rebounded;

$$E_0 = \frac{1}{2}Ux_0^2, \quad E_R = \frac{1}{2}Ux_R^2 \tag{2}$$

The proportion value of the initial displacement of the spring and the displacement after rebound are represented as the rebound number r as Equation (3).

$$r = \left(\frac{x_R}{x_0}\right) = \sqrt{1 - \frac{E_P}{E_0}} \tag{3}$$

From Equation (3), the plastic deformation energy is

$$E_P = E_0\left(1 - r^2\right) \tag{4}$$

The idea of the Brinell hardness was introduced in the classical rebound hammer theory. Equation (5) is the definition of the Brinell hardness, where F, R, d are the working force on the concrete surface, the radius of the contact point and the penetration depth of the hammer tip into the concrete surface respectively.

$$H_B = \frac{F}{2\pi Rd} \tag{5}$$

Then the energy consumed for plastic deformation of the concrete surface is defined as Equation (6).

$$E_P = \frac{1}{2}Fd = \pi Rd^2 H_B \tag{6}$$

Therefore, if both of the penetration depth of the plunger tip d and the rebound number r are measured, the Brinell hardness of the concrete can be determined. However, the only rebound number is measured by the rebound hammer method.

The plastic deformation energy can be defined in another formula as Equation (7).

$$E_P = \frac{F^2}{2\pi RH_B} \tag{7}$$

If the driving force F can be determined, the Brinell hardness of the concrete was given by measuring the rebound number only. As the relationship between the maximum driving force and the initial kinematic energy of the hammer, Equation (8) was deduced.

$$F = \frac{\sqrt{2E_0 K_E}}{\sqrt{X}} \tag{8}$$

Where, K_E is the elastic spring coefficient of the concrete surface and X is

$$X = \frac{E_0}{E_0 - E_P} \tag{9}$$

Let the initial speed of the hammer be V_0 and the mass of the hammer be m, the initial energy of the hammer is

$$E_0 = \frac{1}{2}mV_0^2 \tag{10}$$

Then Equation (8) becomes

$$F = \sqrt{\frac{mK_E}{X}} \cdot V_0 \tag{11}$$

In the classical rebound hammer theory, the spring coefficient of the concrete is assumed as the constant value. If this assumption stands good, using Equation (7), (8) and (9), the Brinell hardness of the concrete can be determined.

Theory of direct driven, plastic body. Let concrete be the plastic body, and the hammer be having spherical crown of radius R. Considering the force equilibrium, Equation (12) is obtained.

$$m\frac{d^2x}{dt^2} + 2\pi R H_B x = 0 \tag{12}$$

Here, x is penetration depth of the hammer. Solving this equation with the initial condition

$$\left.\frac{dx}{dt} = V_0 \right|_{t=0} \tag{13}$$

The maximum force of the hammer is

$$F_{\max} = \sqrt{2\pi R H_B m} \cdot V_0 \tag{14}$$

And the maximum penetration depth is

$$\delta_{\max} = \frac{V_0}{\sqrt{\dfrac{2\pi R H_B}{m}}} \tag{15}$$

As shown in Equation (14) and (15), the maximum driving force and maximum penetration depth are in proportion to the initial velocity of the hammer. And though the maximum driving force is in proportion to the square root of the Brinell hardness and the penetration depth is in inverse proportion of it. It means that the Brinell hardness would be higher, the driving force is to be larger and the penetration depth it to be smaller. Converting Equation (14), the Brinell hardness of the concrete is given in Equation (16).

$$H_B = \frac{1}{2\pi R m}\left(\frac{F_{\max}}{V_0}\right)^2 \tag{16}$$

As shown in Equation (16), determining the Brinell hardness, it is necessary to measure the both of the maximum driving force and the initial velocity of the hammer.

Theory of direct driven, elastic body. Simplified motion equation of the hammer at the elastic deformation process of the concrete is

$$m\frac{d^2x}{dt^2} + K_E x = 0 \tag{17}$$

The initial condition is same as Equation (13). Then the maximum driving force becomes

$$F_{\max} = \sqrt{K_E m} \cdot V_0 \tag{18}$$

Here $\sqrt{mK_E}$ is the definition of the mechanical impedance of the mass-spring system. From the Equation (18) the spring coefficient of the concrete surface is

$$K_E = \frac{1}{m}\left(\frac{F_{\max}}{V_0}\right)^2 \tag{19}$$

The mechanical impedance method to estimate the concrete strength is based on the Equation (19). However, if the elastic deformation process of the concrete surface begins after finishing the plastic deformation process, the initial velocity for the elastic deformation is not equal to the initial velocity of the hammer. Because of the velocity loss by the plastic deformation process may have occurred. To solve this problem, the consideration on the wave form of the hammer acceleration as shown in Figure 1 gives the useful information.

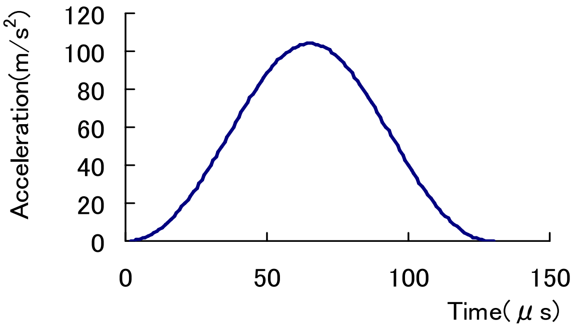

Figure 1 wave form of hammer accelaration

If the function of the concrete surface is as an elastic spring against the hammer blow, the wave form of the hammer acceleration and of the elastic displacement of the concrete surface are similar to each other by the Hook's law. It means when the force becomes maximum value, the elastic displacement of the concrete surface becomes maximum value consequently. The duration when after the driving force reaches the maximum value is the process of that the hammer is pushed back by the concrete surface as the elastic spring. If the concrete surface is perfect elastic body like a spring, the rebound coefficient is 1. This means that the initial velocity for the elastic deformation of the concrete surface is exactly equal to the rebound velocity of the hammer. The rebound velocity of the hammer is calculated as Equation (20).

$$V_R = \int_{T_0}^{\infty} a(t)dt \tag{20}$$

Here, T_0 is the time when the driving force becomes maximum value and $a(t)$ is the acceleration of the hammer. And also the initial velocity when the hammer strikes the concrete surface is

$$V_A = \int_{0}^{T_0} a(t)dt \tag{21}$$

However, the maximum force of the plastic deformation is still unknown. Therefore the Brinell hardness of the concrete can not determined if the rebound number is measured.

On the other hand, the rebound velocity of the hammer is given in Equation (20), substituting V_0 in Equation (19) by V_R of Equation (20) then

$$K_E = \frac{1}{m}\left(\frac{F_{\max}}{V_R}\right) \tag{22}$$

This value is not affected by the plasticity of the concrete surface. The actual mechanical impedance method used this spring coefficient of the concrete to estimate the compressive strength of the concrete.

Stress Wave theory for rebound hammer. The schematic illustration of the simplified structure of the rebound hammer is shown in Figure 2.

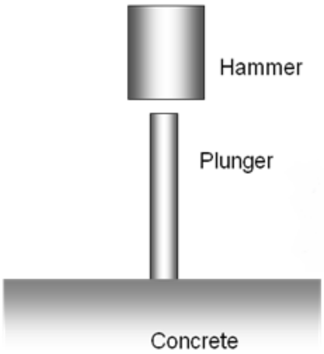

Figure 2 schematic illustration of rebound hammer

The hammer strikes the plunger head, and then the driving force is generated at the plunger head. The driving force propagates to the plunger tip which has contact with the concrete surface. Such a driving mechanism is similar to the pile driving mechanism (Smith 1960). Let the hammer and plunger be an one dimensional elastic bar. The force which is generated at the plunger head is

$$F = \frac{1}{1+\mu} Z_P V_0 \tag{23}$$

Here μ is the mechanical impedance ratio between the hammer and the plunger. Z_P is the mechanical impedance of the plunger. The mechanical impedance of the elastic bar is

$$Z = \frac{AE}{c} \tag{24}$$

Here, A, E, c are the sectional area, Young's modulus and wave speed respectively. And the duration time of the driving force is

$$T = \frac{2m}{Z_H} \tag{25}$$

Here, m, Z_H are mass and the mechanical impedance of the hammer. As shown in Equation (23), the driving force is not affected by any property of the concrete. The force generation mechanism of the direct strike and the indirect drive method are exactly different. And in case of the indirect driving, generated maximum force is defined by the specification of the rebound hammer itself. At the plunger tip, from the force equilibrium, Equation (26) is given.

$$F_{\downarrow}(t) + F_{\uparrow}(t) = F_R(t) \tag{26}$$

Here, $\downarrow, \uparrow$ denotes the direction of the wave propagation, in this case, $\downarrow$ shows the forward wave which propagates from the head of the plunger to the tip of it and $\uparrow$ shows the backward wave which reflected at the plunger tip. F_R is the resistance force of the concrete surface against the plunger tip penetration. The hammer rebound is occurred by the impulse of the backward wave which reaches to the plunger head with compressive wave.

Figure 3 shows the result of numerical analysis (Sakai 1988) on the force at the plunger head. In this analysis, the hammer and the plunger are considered as the one dimensional elastic bar, and the speed of the hammer is used as the initial condition. The spring constant at the plunger and concrete surface contact point is 9.8MN/m as shown in table 1. The rebound value is calculated as the ratio between the impulses of the progress wave before it's becomes tension force and backward wave while it is a compressive force before 100 micro second.

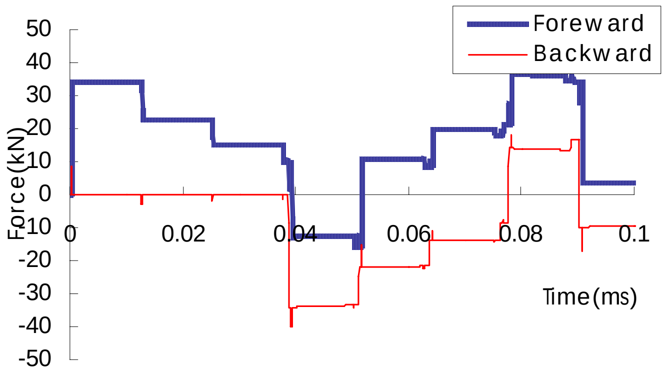

Fig. 3 Result of numerical calculation shown as forces at plunger head

Table 1 Parameters for calculation

element	Sectional area (cm2)	length(cm)	mass(gr)	Initial speed(m/s)
Hammer	15	3.23	380.3	3.4
Plunger	3	10.0	235.5	0.0
Spring constant of concrete MN/m			9.8	

Figure 4 shows the results of the numerical calculations as the relationship between the spring constant of the concrete surface and the rebound number. As shown in Fig. 2, the relationship between them is non linear and if the spring constant of the concrete surface becomes higher, the trend of the rebound numbere is saturated. And also, the rebound value is depended on the elastic characteristics of the concrete.

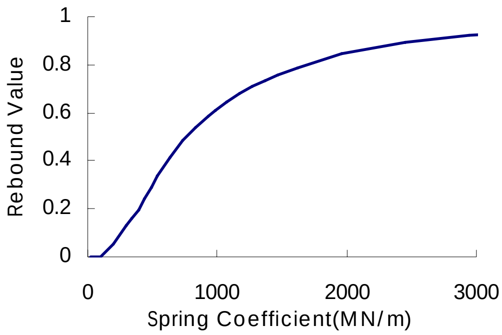

Fig. 4 Relationship between spring constant of concrete surface and rebound value

CONCLUSION

To estimate the compressive strength of the structural concrete in NDT manner, two methods, the rebound hammer method and the mechanical impedance method were examined from the theoretical point of view. In this paper, the fact that the rebound number is related to the elasticity of the concrete surface and it does not represent the plasticity of the concrete surface is shown as the results of the numerical analysis by the stress wave theory. And the maximum driving force of the rebound hammer is limited within the design of the instrument itself, then, it is considered that the applicability for strength estimation of the high strength concrete is limited.

The mechanical impedance method is the method to measure the elastic property of the concrete. If the ultimate strain of the concrete is almost constant for each concrete, the mechanical impedance method gives the high accuracy on the compressive strength estimation as NDT manner.

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