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# Obtaining Rheological Parameters from Slump Flow Test for Self-Compacting Concrete

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## ABSTRACT

Rheological computer simulations of the Abrams cone are introduced in this paper. A Computational Fluid Dynamics software called OpenFOAM (https://www.openfd.o.uk) was used for the calculations. An easy-to-use model for obtaining yield stress and plastic viscosity of concrete on *e.g.* the building site is developed. Promising results show that both yield stress as well as plastic viscosity can be determined by this simple test.

**Keywords.** Numerical Simulation, Self-Compacting Concrete, Slump Flow, Bingham Material Model

## INTRODUCTION

Self-Compacting Concrete (SCC) has since it was first developed in Japan found its way into the precast concrete industry and in many applications on the building sites around the world. Casting without the need for vibrating is an important advantage when it comes to congested geometries of the reinforcement and the formwork, as well as an improvement of the ergonomic aspects for concrete workers. Good performance of SCC includes filling ability of the formwork, passing ability through the reinforcement and resistance to segregation of the concrete mix. The Abrams cone first developed for standard concrete ASTM slump test has now become the most common SCC acceptance test device. It is used to measure slump flow, an important acceptance test to determine concrete workability. Simulations of the ASTM Abrams cone show that the slump flow test also provides information that allows us to derive rheological parameters such as yield stress and plastic viscosity of the concrete, which are usually only obtainable in a lab equipped with a viscometer. Lab tests and simulations of the ASTM mini cone (Tregger *et al.*, 2008), show promising results in predicting rheological parameters for mortars. This paper will focus on an easy-to-use model for obtaining yield stress and plastic viscosity of concrete on *e.g.* the building site. Yield stress and viscosity input simulation values adequately map the self-compacting range suggested by (Wallevik, 2002).

## THEORETICAL ASPECT

**Rheology of Fresh Concrete Flow.** For Pascalian liquids, meaning incompressible fluids (such as concrete) it holds for the fluid velocity vector **u** that

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

The governing equation for non-Newtonian fluids called Cauchy's equation of motion (Malvern, 1969) is given by

$$\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \tag{2}$$

Were **g** is the gravitational acceleration vector acting on the system, $\rho$ is the material density and the stress tensor is $\boldsymbol{\sigma} = -p\,\mathbf{I} + \mathbf{T}$. Here, $p$ denotes pressure, **I** the unit dyadic and **T** is the extra stress tensor, associated with the viscosity of the fluid. For concrete being a viscoplastic material, the relation used for **T** is, (Mase, 1970):

$$\mathbf{T} = 2\eta D\mathbf{u} \tag{3}$$

with $D\mathbf{u}$ being the tensor of rate of deformation as can be found in (Goldstein, 1996):

$$D\mathbf{u} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}) \tag{4}$$

It was shown by (Wallevik, 2003) that Equation (2) is not only applicable for homogeneous fluids, but from a fundamental physical point of view also can be applied on coarse granular systems like fresh concrete. Concrete and other concentrated suspensions are often modelled as a Bingham material. It is a viscoplastic material, showing little or no deformation up to a certain level of stress. For stress levels above the so called yield stress, $\tau_0$, the material flows. In order to fit the Cauchy equation, the apparent viscosity $\eta$ is written as:

$$\eta = \eta_{pl} + \frac{\tau_0}{\sqrt{2D\mathbf{u} : D\mathbf{u}}} \tag{5}$$

where $\sqrt{2D\mathbf{u} : D\mathbf{u}}$ is the shear rate. Equations (1) through (5) define the non-compressible viscosity model used for self-compacting concrete in this paper.

**Spread Diameter.** Assuming that material density and sample volume are known, the final flow spread at flow stoppage is directly correlated to the yield stress of the material. We are also assuming that inertia effects of the spreading flow are neglected for this theory. The slump flow may be treated as a one dimensional problem, since the three-dimensional Abrams cone will reshape into a one-dimensional flow problem with a propagation of much smaller height than its radial spread. This allows one to analyse it analytically. Also, surface tension effects are disregarded since they are much smaller than the apparent viscosity of concrete.

The governing equations for this particular case are based on the physical principles stating that within the system, mass is conserved (the continuity equation) and Newton's second law $F = ma$ (the momentum equation). So starting out with the equation of continuity and the equation of motion for incompressible fluids in cylindrical coordinates (r, θ, z) according to Figure 1, (Kokado *et al.*, 1997) state the equation of continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0 \tag{6}$$

as well as the equation of motion (radial direction r):

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{1}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta}-\frac{\tau_{\theta\theta}}{r}+\frac{\partial \tau_{zr}}{\partial z}\right) \tag{7}$$

Here, $v_r$, $v_\theta$, $v_z$ are the velocity vector components, $\tau_{ij}$ is the stress tensor. Above mentioned simplifications and integration of the equations with the given boundary conditions lead to a relation between the yield stress and flow spread diameter depending on the sample density and volume:

$$\tau_0=\frac{15^2\rho g V^2}{4\pi^2(2R)^5} \tag{8}$$

This relation was found by (Kokado *et al.*, 1997) as well as (Roussel and Coussot, 2005). (Kokado *et al.*, 1997) derived the yield stress by solving the full equations of continuity and equations of motion, whereas (Roussel and Coussot, 2005) described the fluid motion within the long-wave approximation.

**Flow Velocity.** The initial energy amount affecting flow velocity $v$ at cone lifting time t = 0 is depending on the material density, its volume and the height of the cone (*mgh*). As the flow propagates, the speed of flow is associated with fluid viscosity. Depending on the viscosity, a certain stress level of the fluid will result in a particular shear rate, as can be understood from Equation (5).

It is worth noting that inertia effects might affect the final shape of flow once the typical inertia stress (Roussel and Coussot, 2005)

$$I=\rho v^2 \tag{9}$$

Since this value is not considerably (in the order of tenths) lower than the yield stress of the material, as can be seen in section *Yield Stress*.

Once the flow spread can be determined analytically, the dynamics of the cone spread motion may only be captured numerically, using *e.g.* finite element or the like. Nevertheless, in order to grasp the problem and to create a plausible picture of the motion, one will here just scratch the surface of the problem by taking a few steps into it in a simplified way. The following can be said about the moving fluid propagating circular flow:

- The value of the plastic viscosity approaches the measured apparent viscosity as shear rates are increasing, which can be seen in Equation (5)
- The height of the flowing radial part of flow is averaged to $h$
- The shear rate is then approximated as velocity divided by concrete thickness, $v/h$
- The measured viscosity can be approximated as stress τ divided by the shear rate
- The stress τ is given by $\rho g h$

This renders the following relation:

$$\eta_{pl} = A\frac{\rho g h}{\frac{v}{h}} + B \tag{10}$$

with $A(X)$ and $B(X)$ being geometrical parameters, depending on the geometry of the cone and the location of the measuring point $X$. Since we are interested in measuring the frictional (no slippage) flow over most of the flow domain in order to average the flow time over the full concrete spread to obtain higher accuracy, one might be tempted to measure the flow time from cone lifting time $t = 0$ until flow stoppage. However, since it is quite difficult to exactly determine the very moment of concrete (slow motion) flow stoppage, we will choose a different approach. The flow duration will always be timed at a certain given radius $X$ of the flow, *e.g.* $X = 250$ mm, (measuring $T_{500}$). The flow velocity $v$ is now:

$$v = \frac{X}{t_X} \tag{11}$$

However, point $X$ is to be normalized with respect to the total flow radius (travelling length $R$), which gives us the dimensionless measuring point $X/R$. Always using the same measuring point $X$ also ensures the same averaged measuring height $h$ (slump flow thickness) of the same concrete sample volume. Using Equations (10) and (11) with the dimensionless measuring point we can now obtain for a normalized plastic viscosity:

$$\frac{\eta_{pl}}{\rho} = A\frac{g h^2 t_X}{\frac{X}{R}} + B \tag{12}$$

Since the measuring point $x$ and $h^2$ ($X$) are chosen beforehand and always kept constant, they may be treated as constants when comparing obtained values. This is true also for the gravitational acceleration parameter $g$. We would also like to normalize plastic viscosity with respect to specific gravity of the concrete ($\rho_{water}/\rho$). Also, we are aiming at facilitating parameter measurement, which is why the flow spread diameter will be used instead of the radius $R$. Values of interest are now:

$$\frac{\eta_{pl}}{\rho} \cdot \rho_{water} = M \cdot 2R \cdot t_X + N \tag{13}$$

where *e.g.* parameter $M$ now incorporates $0.5\,A$, $g$, $h^2$, $\rho_{water}$ and $1/X$.

## THE NUMERICAL MODEL

The governing equations to be computed are solved by finite volumes. Methods for viscosity and stress are also presented below. It is assumed for the concrete to behave as a homogeneous fluid, no particles are involved in this model.

**Finite Volumes.** Finite Volumes represent a numerical way to solve the partial differential equations used to simulate fluid flow as algebraic statements. The obtained values are calculated on a meshed geometry. Finite volumes refer to a control volume representing a reasonably large, finite region of the flow. Fundamental physical principles are applied to the fluid inside every control volume (Wendt, 1992). In this piece of work, Volume of Fluid, VOF, method is employed as the interface tracking method for the multiphase model (air and concrete). VOF (Hirt and Nichols, 1981) tracks the interphase using a phase marker $\gamma$ such that in a control volume with $\gamma = 0$, only phase one is represented and with $\gamma = 1$, only phase two is represented. $0 < \gamma < 1$ represents an interface in the control volume. The fundamental fluid physical properties vary in space according to the volume fraction of each phase:

$\eta = \eta_1\gamma + \eta_2(1-\gamma)$ and $\rho = \rho_1\gamma + \rho_2(1-\gamma)$

Every cell holding a $\gamma$ value carries a marker, *e.g.* a distinct colour.

**Papanastasiou.** Viscosity given by Equation (5) will render a singular point for zero shear rate (rigid body), which will lead to infinite apparent viscosity. This is avoided by introducing the following equation suggested by (Papanastasiou, 1987):

$$\eta = \eta_{pl} + \frac{\tau_0}{\sqrt{2D\mathbf{u}:D\mathbf{u}}}\left[1 - e^{(-k\sqrt{2D\mathbf{u}:D\mathbf{u}}}\right] \quad (14)$$

The value for $k$ is a very big number, $k = 5000$ for the simulations shown.

**Yield Criterion.** A three-dimensional von Mises yield criterion is used, since the full propagation of flow is to be captured.

## SIMULATIONS

**OpenFoam.** Presented simulations were performed with the downloadable software OpenFOAM, Field Operation And Manipulation (Weller *et al*, 1998). The code is an object-oriented numerical simulation toolkit for continuum mechanics released by OpenCFD Ltd available for free (http://www.opencfd.co.uk). It is a large Continuum Fluid Dynamics library with different types of solvers. This finite volume solver with polyhedral mesh support calculates the mass and momentum equations in their discretized form, which guarantees the conservation of fluxes through the control volume. The code is transparent and may be tailored by the user to fit the problem to be solved.

**Geometry of the Cone.** The Abrams cone has the following dimensions. Its height $h(z)$ = 300 mm, the bottom diameter is 200 mm and the top diameter is 100 mm. The geometry of the cone is shown in Figure 1.

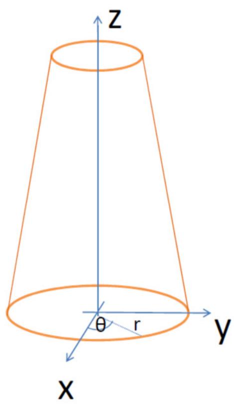


**Figure 1. Geometry of the Abrams Cone**

**Rheological Input Data.** Altogether, 14 different simulations of flow were performed with different density, yield stress and viscosity as shown in Table 1.

**Table 1. Simulated Rheological Input Data of the Concrete**

| **Density [kg/m$^3$]** | **Yield Stress [Pa]** | **Viscosity [Pa·s]** |
|---|---|---|
| 2100 | 30 | 10 |
| 2100 | 45 | 40 |
| 2100 | 50 | 50 |
| 2250 | 10 | 50 |
| 2250 | 15 | 90 |
| 2250 | 20 | 20 |
| 2250 | 40 | 10 |
| 2250 | 40 | 30 |
| 2250 | 70 | 10 |
| 2250 | 70 | 20 |
| 2250 | 90 | 15 |
| 2400 | 5 | 30 |
| 2400 | 10 | 130 |
| 2400 | 45 | 40 |

Yield stress and viscosity input simulation values adequately map the self-compacting range suggested by (Wallevik, 2002).

## RESULTS AND DISCUSSION

A high correlation factor $R^2$ was achieved for the simulated yield stress and viscosity relations.

Both yield stress as well as plastic viscosity are normalized with respect to the specific gravity of the material, rendering $\tau_0 \cdot (\rho_{water}/\rho)$ and $\eta_{pl} \cdot (\rho_{water}/\rho)$. This allows one to compare different concrete mixes.

**Yield Stress.** The relation between the yield stress and the final spread length at flow stoppage is plotted in Figure 2. Simulated values are compared to theoretical values. The simulation trend line ($y = 5148.3e^{-9x}$) is almost coinciding with the theoretical solution. The correlation factor $R^2$ is 0.92, simulated values of the final slump flow are differing more for smaller slumps. Values found above the trend line are of viscosity values below 30 Pa s, the values found below the trend line are simulation with a viscosity higher than 30 Pa s. This leads to the assumption that viscosity, after all, does play a role for the slump flow. Inertia effects cannot be fully disregarded, since the flow propagation may be divided into two different types of flow. A faster, initial flow, followed by a collapsed cone flow, which is a radial and slower, propagating flow. Comparing this fact to Equation (9), with $\rho$ = 2250 kg/m$^3$, a travelling distance of (height of cone + average final radius)/2 $\approx$ (300 mm + 330 mm)/2 = 315 mm and an average spreading time for different types of concrete of about 3 seconds, an inertia effect of about 25 Pa. As seen in Table 1, this is not considerably lower (in the order of tenths) than the material yield stress.

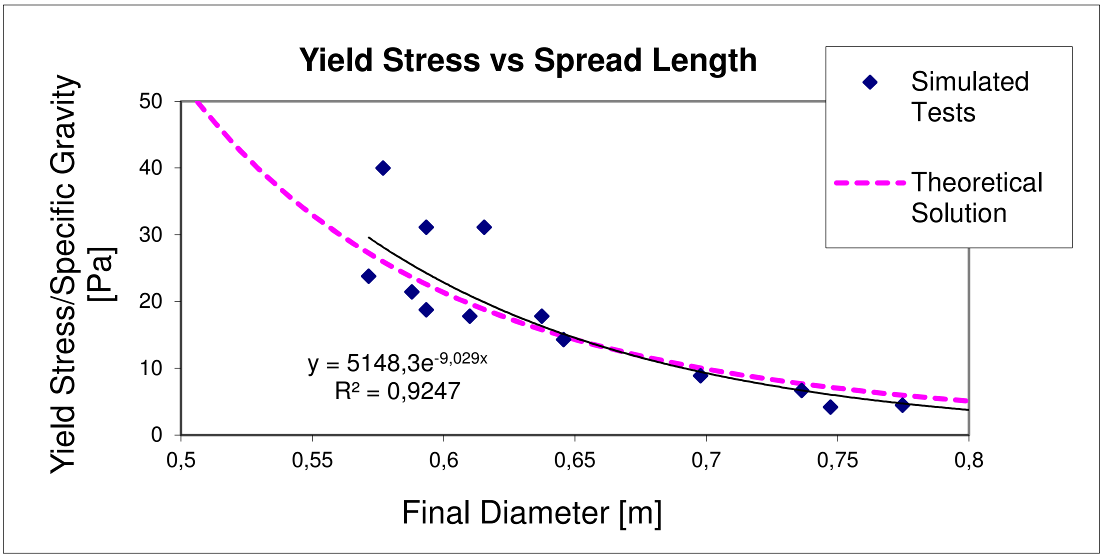


**Figure 2. Output Data for Yield Stress**

As can be seen, these effects are negligible for larger spreads, where the simulated region is closer to the theoretical solution.

**Viscosity.** A linear relation for the plastic viscosity of the simulated concrete was found according to Figure 3. One may detect a link between plastic viscosity and the product of the final spread and $t_X$. Four different measuring points are presented, $X$ = 450, 500, 550 and 600. The best correlation factor is given by $X$ = 600. This is related to the fact that the slower, radial flow is closer connected to the plastic viscosity and is not dominated by inertia effects. However, not all slump flows will reach a diameter of 600 mm. For this reason, a measuring point of $X$ = 550 is suggested. The correlations factor is still high, a close enough value is better than none.

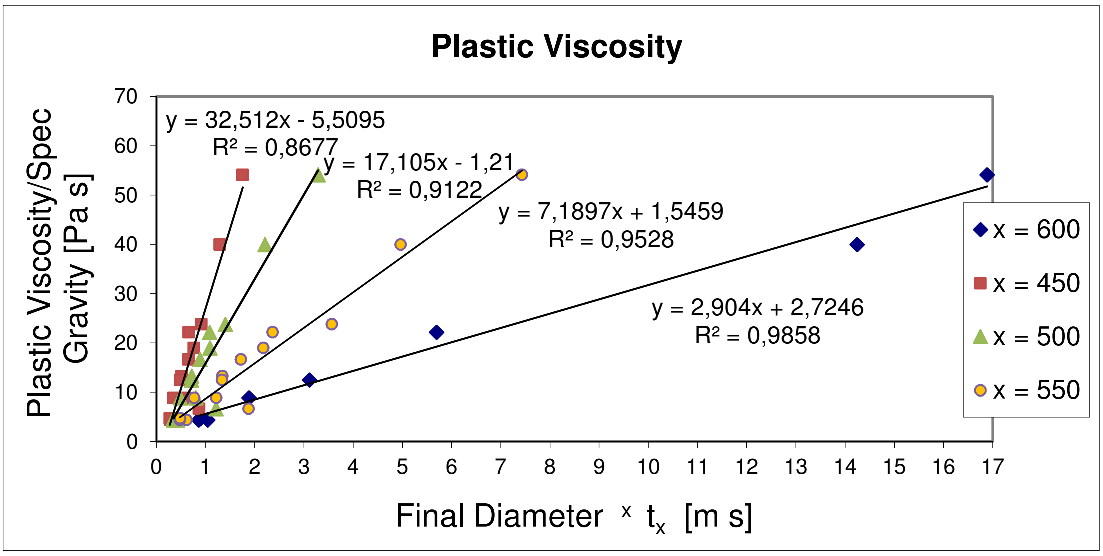


**Figure 3. Output Data for Viscosity**

The trend line for $X$ = 550 is given by y = 7.2x + 1.5. This relation may be compared to Equation (13), linking the normalized plastic viscosity (y) to the product of the final diameter $2R$ and time $t_X$, here simply denoted x.

## CONCLUSIONS

It is quite possible to obtain rheological values with a simple and widely spread testing device, the Abrams cone. Both slump flow and the regular $T_{500}$ are affected by inertia effects, which is why a different measuring point is suggested, $T_{550}$. Once the plastic viscosity has been determined, it is also possible to judge for an over or under estimation of the yield stress according to Figure 2. Trend lines may be developed for high and low viscosity type of concretes. This way, the slump flow value may be adjusted for a more accurate yield value.

This process is of course facilitated using a film camera to capture the flow of the actual (not simulated) concrete, or by a computerized testing device to register the concrete flow propagation.

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