

A Preliminary Study on the Effect of Separate Addition of Lignosulfonate Superplasticiser and Waterglass on the Rheological Behaviour of Alkali –activated Slags

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ABSTRACT

The poor workability of Alkali-activated slag (AAS), a novel non-Portland cement (PC) hydraulic material, has hindered its wider industrial applications. Current commercial superplasticisers are mainly designed for PC, which are dysfunctionalised in AAS due to the interaction with the alkaline activator in AAS. To reduce this interaction, adding SP and activator separately would be a potential method. In this paper, the effects of separate addition of lignosulfonate superplasticiser (LS SP) and waterglass on the rheological behaviour of waterglass-activated slags were investigated. The results showed that the minislump spreads of AAS increased with increasing LS SP dosage and it was also observed that the minislump spreads increased by adding LS SP and alkaline activator separately. It was showed that the shear thickening behaviour occurred when the LS SP and the activator were added separately. Bingham, Modified Bingham and Herschel-Bulkley model were employed to describe those rheological behaviour and the results suggested the Modified Bingham and Herschel-Bulkley model were better suited than Bingham model for describing the rheology of AAS system. It was clearly showed that the separated addition could highly reduce both yield stress and plastic viscosity (consistence factor in the case of Herschel-Bulkley model).

Keywords Different addition, Rheological properties, Shear Thickening, Lignosulfonate Superplasticiser, Alkali-activated slag paste

INTRODUCTION

Rheological properties are critically important for cement and concrete in order to achieve desirable mechanical and durability properties. It is essentially a period of plasticity of fresh concretes required for the process of transporting, placing, compacting and finishing. The rheological properties of Portland cement (PC) - based systems, have been widely studied, and most of the results showed that fresh PC concrete is a thixotropic material which is featured by a decrease in viscosity when certain amount of shear is applied and followed by a gradual recovery when shear is removed. Although it is generally agreed that the down ramp of hysteresis loop of the thixotropic materials can be well fitted by almost any

rheological models (Banfill, 1994), the most widely accepted is the Bingham model (Equation 1) (Tattersall and Banfill, 1983, Banfill, 1994). The yield stress, which is the intercept from the Bingham model, can be considered as the transition point below which the substance behaves as a solid and above it becomes fluids (Lewis et al., 2000), resulting from the attractive interparticle forces responsible for the flocculation (Banfill, 1994, Barnes and Walters, 1985). Thus, substance with a lower yield stress reflects a better dispersion and fluidity. On the other hand, the plastic viscosity, the slope from the Bingham equation, depends largely on the volume friction of solid particles and the packed density (Struble and Lei, 1995). As a result, low plastic viscosity might cause the segregation (Feys et al., 2008).

However, with the advent of self-compacting concrete (SCC), especially with the addition of superplasticisers, the yield stress of SCC is much lower than that of traditional concretes in order to achieve much improved fluidity (Flatt, 2004, De Schutter et al., 2008, Feys et al., 2008). In some cases, in particular in the presence of thickeners and higher dosage of superplasticisers, the negative yield stress and a non-linear shear thickening (in which case the plastic viscosity increased with the increase of shear rate) have been identified (Larrard et al., 1998, Cyr et al., 2000). As a result, the modified Bingham model (Equation 2) and the Herschel-Bulkley model (Equation 3) are usually used to describe this shear thickening rheological behaviour (Feys et al., 2008, Cyr et al., 2000, Nguyen et al., 2011).

$$\tau = \tau_0 + \mu \cdot \dot{\gamma} \quad (1)$$

$$\tau = \tau_0 + \mu \cdot \dot{\gamma} + c \cdot \dot{\gamma}^2 \quad (2)$$

$$\tau = \tau_0 + K \cdot \dot{\gamma}^n \quad (3)$$

Where:

τ stands for shear stress (Pa); τ_0 yield stress (Pa); μ plastic viscosity (Pa s); $\dot{\gamma}$ shear rate (s^{-1}); K consistency factor ($Pa \cdot s^n$); c second order parameter ($Pa \cdot s^2$), and n exponent (-).

Recently, alkali-activated slag binder (AAS), has received increased attention worldwide due to its sustainability nature (Shi et al., 2006). The AAS mainly consists of alkaline activators (such as sodium hydroxide and sodium silicate) and ground granulated blast furnace slag (GGBS), a by-product from iron and steel manufacture. However, the poor workability of AAS becomes one of the main barriers hindering its wider industrial applications. To solve these issues, superplasticisers (SP) have to be employed to improve its fresh properties in a similar way for PC-based system. (Bakharev et al., 2000). It has been reported by different researchers that adding the plasticisers or superplasticisers together with the alkaline activators were unable to reduce the yield stress of waterglass-activated slag pastes, presumably due to the interactions between the plasticisers/superplasticisers with the activators under highly alkaline environment (Palacios et al., 2009, Palacios et al., 2008). This dysfunction of PC-based SP could be due to the increased chemical instability and complex competitive adsorption between the SP and activator when adding them at the same time. Previous work carried out by the authors (Ren et al., 2012) indicated that by adding PC-based SP and the activator separately, both the adsorption of PC-based SP and the fluidity of AAS paste could be improved. However, the rheological behaviour of AAS prepared by the separated addition method was not well studied.

The aim of this study is therefore to obtain a better understanding on the rheological behaviour of AAS in the presence of PC-based lignosulfonate derivation superplasticiser (LS SP) with different addition methods. Attempts were then made to use the Bingham model,

Modified Bingham model and Herschel-Bulkley model to describe the change to the rheological behaviour caused by different addition methods. Meantime, the influence of different addition methods on the dispersion properties of fresh AAS pastes was also examined by mini-slump. The suitability of different models for describing the rheological behaviour of waterglass-activated AAS is also discussed.

EXPERIMENTAL

Materials

Ground granulated blast furnace slag (GGBS) used in this paper was supplied by Civil and Marine Ltd. UK (Table 1). The activator was liquid sodium silicate, which was modulated to a modulus of 1.5 by adding sodium hydroxide (NaOH). Both raw liquid sodium silicate and NaOH were obtained from Charles Tennant & Co Ltd and Tennants Distribution, respectively. Solid sodium lignosulfonate derivation superplasticiser (LS SP) was supplied by Tianjin Jiangong Special Material Co. Ltd.

Table 1 Chemical composition of GGBS

| % | CaO | SiO ₂ | Al ₂ O ₃ | MgO | Sulphide | TiO ₂ | Mn ₂ O ₃ | Na ₂ O | Fe ₂ O ₃ | K ₂ O | LOI |
|------|-------|------------------|--------------------------------|------|----------|------------------|--------------------------------|-------------------|--------------------------------|------------------|------|
| GGBS | 39.40 | 34.30 | 15.00 | 8.00 | 0.80 | 0.70 | 0.50 | 0.45 | 0.40 | 0.38 | 0.70 |

Mixing procedures

The GGBS (slag for short hereafter) was activated by waterglass, which was modulated to a modulus of 1.5 by sodium hydroxide, and added at a total alkali content of 4% Na₂O equivalent by mass of the slag. The water to slag ratio of all the mixes was fixed at 0.45. The dosage of LS SP was controlled at 0%, 0.4%, 0.8%, 1.2%, 1.6%, and 2.0% (by the mass of slag). Both the activator and LS SP were pre-dissolved in water. Three different addition methods were studied, namely: 1) *simultaneous addition (SA)*: adding SP and activator together and then mixing with slag; 2) *prior addition (PA)*: adding SP to slag first (with 2/3 of the total mixing water), then activator at 3 min interval; and 3) *delayed addition (DA)*: adding activator to slag (with 2/3 of the total mixing water) first and then SP at 3 min interval.

Test Procedures

The rheological behaviour of alkali-activated slag pastes, with and without lignosulfonate, was determined with previously mentioned three different addition methods. The same paste was also used to determine the minislump spread. The total mixing time of all the mixes was fixed constantly at 6 minutes. Immediately after mixing, the pastes were transferred into the container of a rheometer, Viscotester 550, which uses a helical impeller to establish the relationship between different shear stress and shear rate in order to obtain the fundamental information needed for fitting the rheological models as described by equations 1 and 2 in this study. The samples were subjected to a cycle measuring procedure following Palacios (Palacios et al., 2008). During the cycle, the shear rate was kept constant at 150 s⁻¹ for 2 minutes for preshearing, then up-ramped from 0 to 10 s⁻¹ for 1 min, continually raised from 10 to 130 s⁻¹ in 1 min and finally reduced from 130 to 0 s⁻¹ in 1 min. The results were recorded by software. The same paste used in the rheology test was also used for the mini-slump test, carried out with a PVC plate and a cone with a lower inner diameter of 38.1 mm, an upper inner diameter of 19 mm, and a height of 52.7 mm. The diameters of the spread

from the mini-slump test were measured at two perpendicular directions and the average diameter was reported. All the mini-slump measurements were conducted at 7 minutes after mixing.

RESULTS AND DISCUSSION

Flow curve. A typical rheological flow curve of waterglass-activated slag pastes obtained from different addition methods at 2.0% dosage of LS SP is shown in Figure 1. The curve of the control mix (i.e. the waterglass-activated slag without SP) is also presented. It can be seen from Fig.1 that, without superplasticiser, the hysteresis loop of the waterglass activated slag paste was larger than those of the AAS pastes with SP. These results suggest that the paste without SP is better structured than those with SP, indicating that the shear energy imparted during mixing and preshear is insufficient to break down the structure of

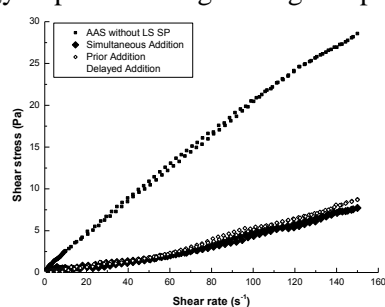


Figure 1 A typical flow curve of waterglass-activated slag with different SP addition methods (2.0% LS SP dosage by the mass of slag)

pure AAS. This could be due to the slag flocculation and the formation of the primary C-S-H gel (Palacios et al., 2008). From Figure 1, it is also evident that not only the addition of SP, but also its addition method, could change the down-curve of the paste. The down-curve of both the pastes mixed without SP and mixed by SA method showed a near-linear relationship between shear stress and shear rate, while those mixed by PA and DA methods showed a power law relationship. Similar patterns were also identified at other dosage levels (0.4%, 0.8%, 1.2% and 1.6%).

Rheological model. A typical fit curve of the pastes with 2.0% dosage of LS SP is shown in Figure 2 (a) to (d). It is clear that the fit-curves of these three rheological models were close to each other for the reference mix (i.e. mix with no LS SP). However, the fit curve of the Bingham model was slightly differed from those of the other two non-linearity models (Modified Bingham and Herschel-Bulkley) with the addition of LS SP. And the difference of the fit-curve between Bingham and non-linearity models was even further enlarged by separated addition. It could be easily found from Figure 2 (c) and (d) that the regression curves of both non-linearity models are better fitted than that of Bingham model. And negative yield stress is only obtained by applying the Bingham model. In addition, all the down curves of the pastes mixed with SA, PA and DA methods at the other LS SP dosages are also fit into the Bingham, Modified Bingham and Herschel-Bulkley models (the regression equation and R-squared value are listed in Table 2). It is obvious that good regressions ($R^2 > 0.9$) have been achieved by all three rheological models. However, by adding LS SP and the activator separately, reduction in R^2 value is occurred in the Bingham model, indicating the reliability of the Bingham model for the separated addition methods might have been reduced.

Yield stress. Since shear thickening behaviour with a non-linear relationship between the shear stress and shear rate was identified from all the mixes with separate addition of SP and activator, the Bingham (linear), Modified Bingham (non-linear) and Herschel-Bulkley (non-linear) models were, therefore, introduced and compared in greater detail in this paper. In the presence of LS SP, the yield stresses of waterglass-activated slag pastes calculated from the

Bingham (Figure 3(a)), Modified Bingham (Figure 4(a)) and Herschel-Bulkley models (Figure 5(a)) are reduced with the increase of SP dosage, especially so when adding SP and activator separately (both *PA* and *DA*). Compared to the *SA* method, the separate additions highly reduced the yield stress and a higher reduction was achieved by *PA*. However, some negative yield stresses from all the mixes by *PA* and *DA* can be observed in the regression using the Bingham model, which could not happen in reality and also conflicts with the common rheology science. These negative yield stresses also raise the doubt over the suitability of the Bingham model for describing the rheological behaviour of AAS with LS SP being added by *PA* and *DA* methods. The reason for the negative yield stress could be due to the change of the rheological properties from near Bingham to shear thickening through the separate additions. From the regression equation summarised in Table 2, it also can be seen that the intercept of the equations from all three methods is reduced with increasing LS SP dosage, indicating a reduction of the yield stress of AAS pastes. Meantime, the intercept of the equation is also reduced from *SA* method to *PA* and *DA* method. These results suggested that compared with the simultaneous addition method, the reduction of yield stress from the separated additions was much larger, and the lowest value was observed from the *PA* method.

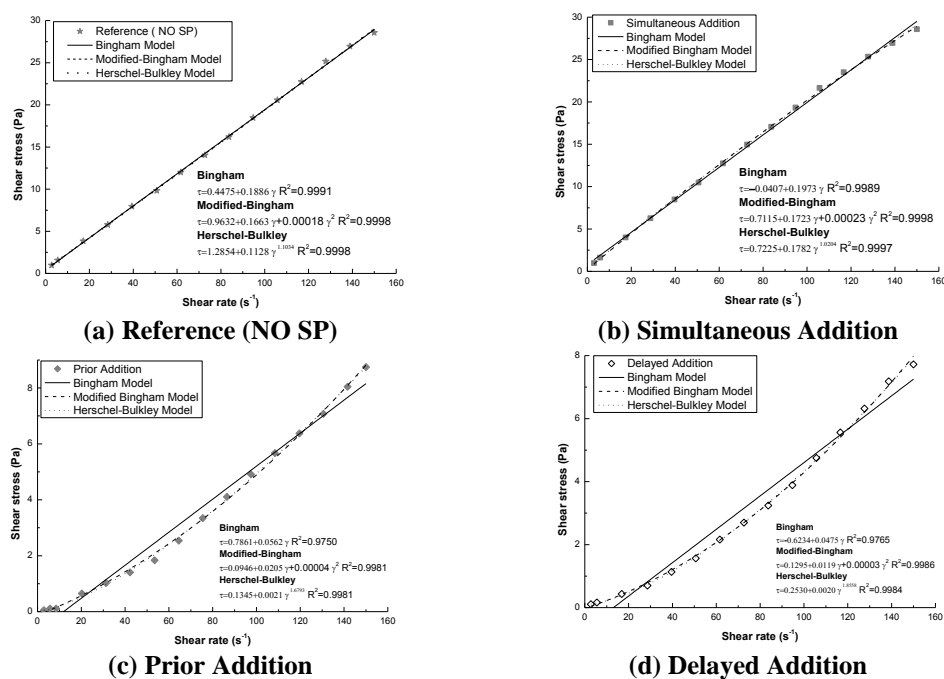
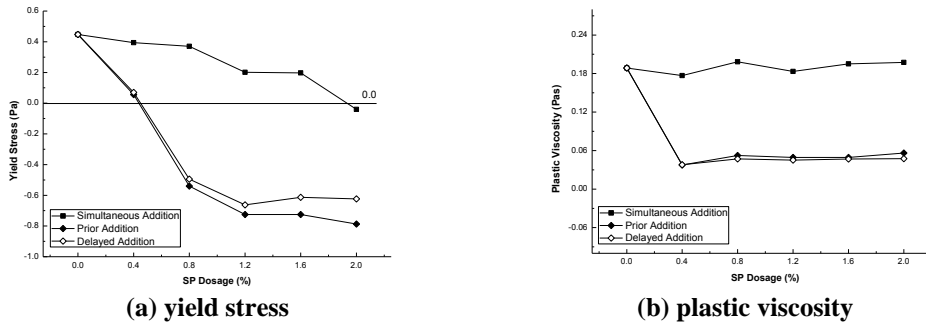


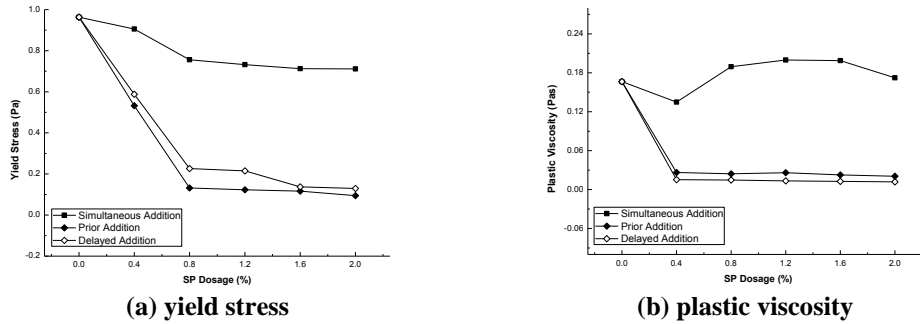
Figure 2 Effect of different addition methods on rheological properties (down curve) of waterglass-activated slag at 2.0% LS SP dosage by the mass of slag

Plastic viscosity and consistence factor. The effect of different LS SP addition methods on the plastic viscosity (based on the Bingham model and the Modified Bingham Model) of waterglass-activated slag pastes is plotted in Figures 3 (b) and 4 (b). It is obvious that increasing the SP dosage had less effect on the plastic viscosity for the *SA* method. In fact, slight increase in the plastic viscosity can be observed from both Bingham and Modified Bingham Models. When adding LS SP and waterglass separately, the plastic viscosity was significantly decreased in both models. However, there was less difference between the *PA* and the *DA* methods. On the other hand, the effect of different addition methods on the consistence factor (based on the Herschel-Bulkley model) is different. From Figure 5(b), it can be seen that there is no obvious trend of consistence factor with increasing SP dosage by

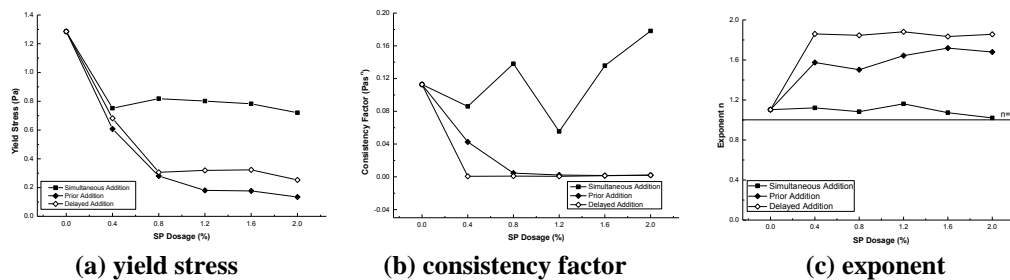
simultaneous addition (SA method). However, the consistence factor obtained from both the *PA* and the *DA* methods was lower than that from the *SA* and a lower value was found from the *DA* at the dosage levels lower than 0.8%. Beyond 0.8% dosage, a plateau was reached for both *PA* and *DA* methods and the change of LS SP dosage had no effect on the consistence factor thereafter.



(a) yield stress (b) plastic viscosity
Figure 3 Effect of SP dosage added by different methods on rheological properties of waterglass-activated slag based on Bingham model



(a) yield stress (b) plastic viscosity
Figure 4 Effect of SP dosage added by different methods on rheological properties of waterglass-activated slag based on Modified Bingham model



(a) yield stress (b) consistency factor (c) exponent
Figure 5 Effect of SP dosage added by different method on rheological properties of waterglass-activated slag based on Herschel-Bulkley model

Exponent (only for the Herschel-Bulkley model). The exponent from the Herschel-Bulkley model (Equation 3) for the waterglass-activated slag paste is plotted in Figure 5 (c). It is clearly showed that the exponent of the reference AAS paste (i.e. mix with no LS SP) was around 1.0 and the addition of LS SP by the *SA* method had less effect on the exponent value. However, the exponent increased when the LS SP and activator were added separately, indicating a shear thickening rheological behaviour at both the *DA* and the *PA* methods. It should be noted that a higher exponent was achieved by the AAS pastes mixed by the *DA* method.

Minislump. The results of the initial mini slump tests for the AAS pastes in the presence of LS SP are shown in Figure 6. It is obvious that adding LS SP could increase the minislump spread of waterglass-activated slags and the fluidity increased with increasing dosage of LS SP. However, the increase in the minislump from the simultaneous addition was less significant than those from the separate addition methods. The spread diameter of mini slump obtained from both prior and delayed addition methods was at least 10 mm higher than that of the SA method, which correlates well with the yield stress results.

It has been established by various researchers that a correlation existed between slump and yield stress. To certain extent, this correlation is somehow expected, because in the slump test, the rate of the movement of concrete is very small and the concrete is at rest when the slump is measured. Therefore, it could be considered that the workability test is approximate to a special two-point test where the shear rate is zero or near zero, which will establish the theoretic basis where the slump result could be correlated with the yield stress. In the current study, a relationship between the yield stress obtained from all the results and the minislump spread has also been identified based on three different models as shown in Figure 7. An equation as $\tau = -7.339\ln(x) + 35.450$ with the coefficient of 0.88 was obtained from Bingham model (Figure 7(a)); $\tau = -5.226\ln(x) + 25.842$ with the coefficient of 0.92 was obtained from Modified Bingham model (Figure 7(b)); and $\tau = -5.028\ln(x) + 24.964$ with coefficient of 0.86 was obtained from Herschel-Bulkley Model (Figure 7(c)). The results suggest that among the three models, the Modified Bingham model can give a better indication of the situation of the yield stress as measured by mini-slump test. However, there is no relationship between the plastic viscosity (consistency factor) and the minislump with the change of SP dosage (Figure 8(a), (b) and (c), which is as expected based on the findings reported in PC systems.

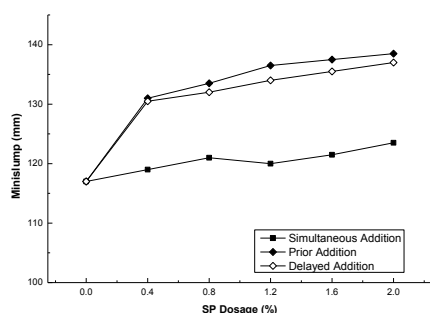
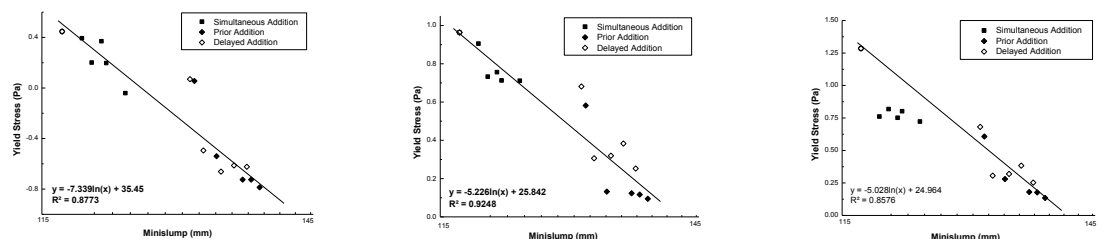


Figure 6 Effect of different addition methods on initial mini slump results of waterglass-activated slag at 3 min interval

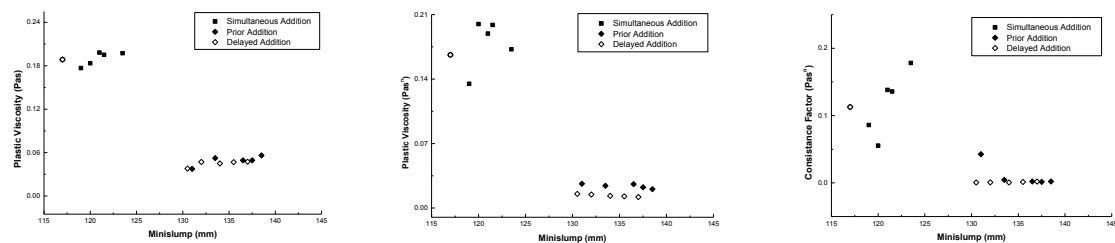
Rheograph. The effects of the LS SP dosages on both yield stress and plastic viscosity (consistency factor) by different addition methods are shown in Figure 9, which is presented by following the vectorised-rheograph approach (Wallevik and Wallevik, 2011). Comparing the plots based on Bingham (Figure 9(a)), Modified Bingham (Figure 9(b)) and Herschel-Bulkley (Figure 9(c)) models, the trend and the distribution of the results from these three models are all very similar, except those from Herschel-Bulkley model with simultaneous addition which are randomly distributed without showing clear trend. These results clearly demonstrated that by adding the LS SP and the activator separately, both the yield stress and the plastic viscosity have been effectively reduced, resulting in the redistribution of the results to the bottom left of the rheograph. In addition, with the increase of LS SP dosages, improved plasticizing effect (except 0.4% of LS SP dosage) can be observed from all the methods, although it is insignificant for the simultaneous addition method. Comparing the *PA* method with the *DA* method, a lower yield stress can be observed from the *PA*, while a lower plastic viscosity is obtained from the *DA*.

In Portland cement-based cementitious system, Bingham model (Equation (1)) has been widely accepted for describing the rheological behaviours of different matrices. However, in the current study, the employment of Bingham model in the AAS mixes under separate addition methods has led to negative yield stress in some mixes, which contradicts with the

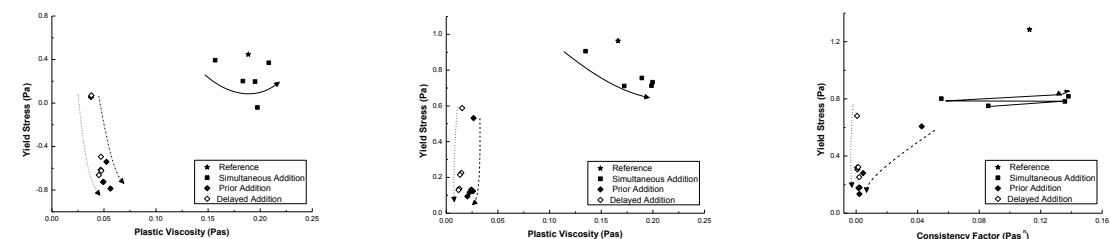
common science of rheology. Similar behaviour have been observed from SCC and it has been reported that, instead of linear fitting based on the Bingham model, the nonlinear fitting with Modified Bingham and Herschel-Bulkley models can be used to describe these rheological behaviour without introducing negative yield stress value. In our study, same conclusions could be made from the Modified Bingham and Herschel-Bulkley models, demonstrating their better suitability for describing AAS-based systems. However, the understanding of the second order term in the context of rheology science is still to be explored.



(a) Bingham model (b) Modified Bingham model (c) Herschel-Bulkley model
Figure 7 Relationship between initial minislump spread and yield stress



(a) Bingham model (b) Modified Bingham model (c) Herschel-Bulkley model
Figure 8 Relationship between initial minislump spread and plastic viscosity



(a) yield stress Vs. plastic viscosity by Bingham model (b) yield stress Vs. plastic viscosity by Modified Bingham model (c) yield stress Vs. consistency factor by Herschel-Bulkley model

Figure 9 Effect of different addition method on the rheograph for waterglass-activated slag paste (The direction of arrow indicates the increase of LS SP dosage)

CONCLUSION

Based on the results presented in this paper, the following conclusions can be drawn:

- (1) Compared to simultaneous addition method, separate addition methods changed the rheological behaviour from Bingham-like fluid to shear thickening.
- (2) In waterglass-activated slag paste, the spread diameter of mini-slump obtained from prior and delayed addition methods was higher than that of the simultaneous addition method, with the prior addition better than the delayed addition.

- (3) The effect of different addition methods of LS SP on waterglass-activated slag was described on a rheograph using yield stress against vs. plastic viscosity. The yield stress and plastic viscosity (consistence factor) obtained from all Bingham, Modified Bingham and Herschel-Bulkley models of waterglass-activated slag pastes were reduced by adding the SP and the activator separately. The Bingham model did not fit well the waterglass activated slag in the presence of SP due to the negative yield stress obtained. Non-linearity model (Modified Bingham and Herschel-Bulkley model) were identified to be a better model for describing the effect of LS SP on rheological properties of waterglass activates slags.
- (4) Whilst the effect of different addition methods was investigated, the mechanism of this different addition method has not been exploited yet. Further study is still needed to understand the mechanisms involved.

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Table 2 Regression analysis based on different rheological models

| Addition Method | SP Dosa ge | Bingham Model | | Modified Bingham Model | | Herschel-Bulkley Model | |
|-----------------------|------------|--|----------------|---|----------------|--|----------------|
| | | Equation | R ² | Equation | R ² | Equation | R ² |
| Reference | 0 | $\tau = 0.4475 + 0.1886 \cdot \dot{\gamma}$ | 0.9991 | $\tau = 0.9632 + 0.1663 \cdot \dot{\gamma} + 1.8545E-04 \dot{\gamma}^2$ | 0.9998 | $\tau = 1.2854 + 0.1128 \cdot \dot{\gamma}^{1.1034}$ | 0.9998 |
| Simultaneous Addition | 0.4% | $\tau = 0.3941 + 0.1768 \cdot \dot{\gamma}$ | 0.9989 | $\tau = 0.9054 + 0.1349 \cdot \dot{\gamma} + 1.8192E-04 \dot{\gamma}^2$ | 0.9998 | $\tau = 0.7608 + 0.0860 \cdot \dot{\gamma}^{1.1210}$ | 0.9998 |
| | 0.8% | $\tau = 0.3702 + 0.1982 \cdot \dot{\gamma}$ | 0.9997 | $\tau = 0.7560 + 0.1893 \cdot \dot{\gamma} + 2.2713E-04 \dot{\gamma}^2$ | 0.9997 | $\tau = 0.7516 + 0.1381 \cdot \dot{\gamma}^{1.0825}$ | 0.9993 |
| | 1.2% | $\tau = 0.2015 + 0.1834 \cdot \dot{\gamma}$ | 0.9977 | $\tau = 0.7324 + 0.1997 \cdot \dot{\gamma} + 2.5648E-04 \dot{\gamma}^2$ | 0.9995 | $\tau = 0.8181 + 0.0554 \cdot \dot{\gamma}^{1.1616}$ | 0.9992 |
| | 1.6% | $\tau = 0.1972 + 0.1951 \cdot \dot{\gamma}$ | 0.9995 | $\tau = 0.7127 + 0.1987 \cdot \dot{\gamma} + 2.3653E-04 \dot{\gamma}^2$ | 0.9998 | $\tau = 0.8016 + 0.1358 \cdot \dot{\gamma}^{1.0728}$ | 0.9998 |
| | 2.0% | $\tau = -0.0407 + 0.1973 \cdot \dot{\gamma}$ | 0.9989 | $\tau = 0.7115 + 0.1723 \cdot \dot{\gamma} + 2.2574E-04 \dot{\gamma}^2$ | 0.9998 | $\tau = 0.7225 + 0.1782 \cdot \dot{\gamma}^{1.0204}$ | 0.9997 |
| Prior Addition | 0.4% | $\tau = 0.0558 + 0.0375 \cdot \dot{\gamma}$ | 0.9753 | $\tau = 0.5319 + 0.0264 \cdot \dot{\gamma} + 4.8967E-05 \dot{\gamma}^2$ | 0.9942 | $\tau = 0.6075 + 0.0426 \cdot \dot{\gamma}^{1.5744}$ | 0.9934 |
| | 0.8% | $\tau = -0.5403 + 0.0524 \cdot \dot{\gamma}$ | 0.9840 | $\tau = 0.1318 + 0.0242 \cdot \dot{\gamma} + 4.4142E-05 \dot{\gamma}^2$ | 0.9975 | $\tau = 0.2804 + 0.0045 \cdot \dot{\gamma}^{1.5025}$ | 0.9979 |
| | 1.2% | $\tau = -0.7251 + 0.0494 \cdot \dot{\gamma}$ | 0.9760 | $\tau = 0.1232 + 0.0259 \cdot \dot{\gamma} + 4.5804E-05 \dot{\gamma}^2$ | 0.9971 | $\tau = 0.1802 + 0.0022 \cdot \dot{\gamma}^{1.6428}$ | 0.9985 |
| | 1.6% | $\tau = -0.7254 + 0.0493 \cdot \dot{\gamma}$ | 0.9721 | $\tau = 0.1164 + 0.0226 \cdot \dot{\gamma} + 4.0525E-05 \dot{\gamma}^2$ | 0.9971 | $\tau = 0.1771 + 0.0015 \cdot \dot{\gamma}^{1.7182}$ | 0.9975 |
| | 2.0% | $\tau = -0.7868 + 0.0562 \cdot \dot{\gamma}$ | 0.9750 | $\tau = 0.0946 + 0.0205 \cdot \dot{\gamma} + 4.1105E-05 \dot{\gamma}^2$ | 0.9981 | $\tau = 0.1345 + 0.0021 \cdot \dot{\gamma}^{1.6793}$ | 0.9981 |
| Delayed Addition | 0.4% | $\tau = 0.0698 + 0.0380 \cdot \dot{\gamma}$ | 0.9601 | $\tau = 0.5880 + 0.0153 \cdot \dot{\gamma} + 3.7770E-05 \dot{\gamma}^2$ | 0.9995 | $\tau = 0.6815 + 0.0006 \cdot \dot{\gamma}^{1.8601}$ | 0.9953 |
| | 0.8% | $\tau = -0.4944 + 0.0471 \cdot \dot{\gamma}$ | 0.9665 | $\tau = 0.2261 + 0.0147 \cdot \dot{\gamma} + 3.3554E-05 \dot{\gamma}^2$ | 0.9990 | $\tau = 0.3062 + 0.0008 \cdot \dot{\gamma}^{1.8461}$ | 0.9989 |
| | 1.2% | $\tau = -0.6620 + 0.0451 \cdot \dot{\gamma}$ | 0.9617 | $\tau = 0.2150 + 0.0133 \cdot \dot{\gamma} + 3.0604E-05 \dot{\gamma}^2$ | 0.9993 | $\tau = 0.3194 + 0.0007 \cdot \dot{\gamma}^{1.8810}$ | 0.9974 |
| | 1.6% | $\tau = -0.6132 + 0.0468 \cdot \dot{\gamma}$ | 0.9694 | $\tau = 0.1371 + 0.0127 \cdot \dot{\gamma} + 2.9996E-05 \dot{\gamma}^2$ | 0.9967 | $\tau = 0.3830 + 0.0014 \cdot \dot{\gamma}^{1.8339}$ | 0.9971 |
| | 2.0% | $\tau = -0.6234 + 0.0475 \cdot \dot{\gamma}$ | 0.9765 | $\tau = 0.1295 + 0.0119 \cdot \dot{\gamma} + 2.7115E-05 \dot{\gamma}^2$ | 0.9986 | $\tau = 0.2530 + 0.0020 \cdot \dot{\gamma}^{1.8558}$ | 0.9984 |