Advanced Computational Methods and Solutions in Civil and Structural Engineering

By

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The work contained within this document has been submitted
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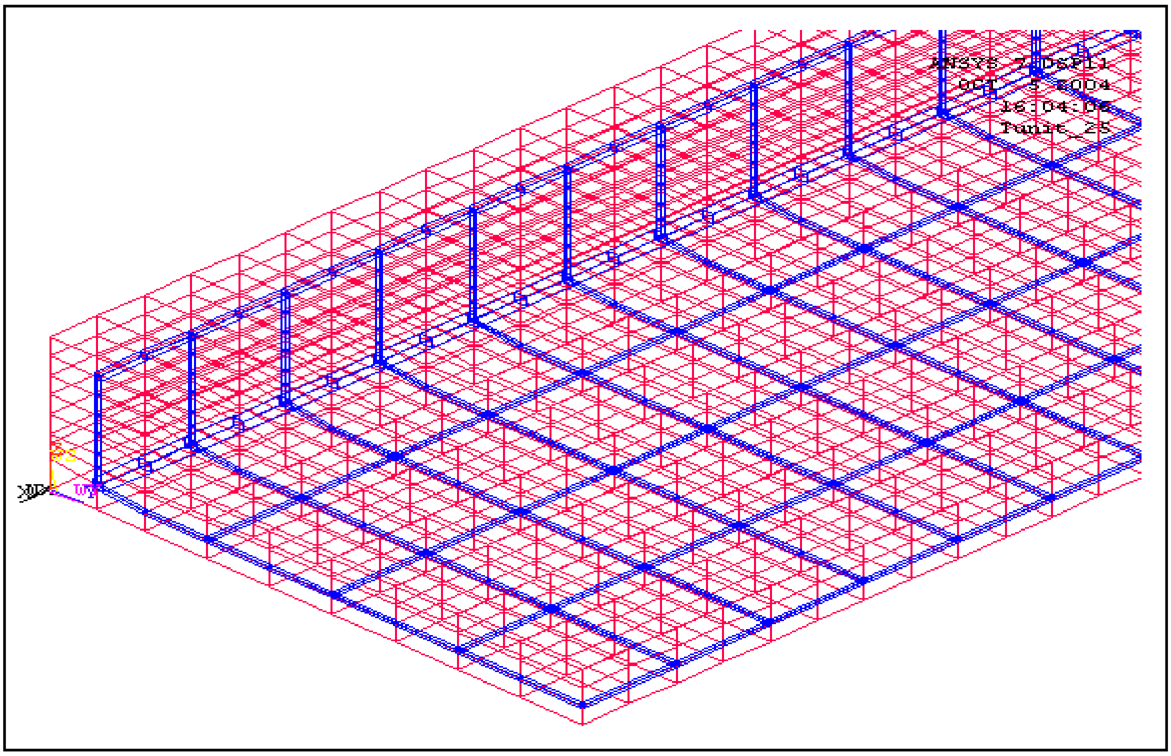

A Portfolio of Published Articles submitted to the Faculty of Engineering and Computing, Coventry University, in partial fulfilment of the requirements for the degree of Doctor of Philosophy (PhD).

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SYNOPSIS

The accurate solution of complex engineering problems has been the objective of a number of generations of engineers and scientists. This has been achieved, to a large extent, by using experimental, analytical (exact) or numerical (approximation) techniques, or any combination of the three.

The author has undertaken long-term research and published in two main areas of Civil and Structural Engineering: Concrete pavement structures and vibrations of concrete grandstands. His work has made a contribution to the subject of numerical analysis of structures, where by employing advance computational (numerical) techniques he has been able to simulate reinforced concrete behaviour successfully and solve complex engineering problems.

In brief, the author has developed a non-destructive method for assessing the structural condition of concrete pavements. He represented soil and other unbound layers of a pavement structure analytically, by introducing a cubic spline and an exponential function in the algorithm. Based on the above, he developed a successful numerical model and used it to evaluate the stiffness of the different layers of a rigid pavement combining the latter with surface deflections.

At a later stage, he was able to model concrete grandstand terraces by developing a general elasto-plastic, constitutive, meso-macro scale level numerical model, featuring cracking and crushing characteristics, as well as yielding for steel reinforcement. His numerical model was capable of simulating the real behaviour of concrete, hence extendable to represent any concrete structure and therefore of practical value to engineering community.

The value of being able to effectively predict the dynamic behaviour of a structure has been long recognised together with the fact that the costs for both experimental and analytical approach have been large. Only large organizations have been able to provide their design engineers with the modern experimental and analytical tools necessary to perform such tasks. The author has been able to show how fully justifiable structural modification procedures, combined with finite element updating, can be used, to improve correlation between experimental and predicted results on an inexpensive desktop computer.

The author has demonstrated his interest in working with other materials outside concrete, by developing a successful finite element model, this time of a structural steel connection. He introduced materials as well as geometric non-linearities (large deformation effects) in the analysis procedure and was able to improve the initially predicted results. He recommended his procedure to other researchers and practitioners interested on accurate representation of similar structures.

In concluding, the author has made a sustained and systematic contribution to engineering knowledge and practice resulting also in his own development. However, he is fully aware of the limitations and drawbacks of his work and he endeavours to continue to minimise and improve it and make a further contribution to the subject by continuously seeking new, more accurate and innovative ways to solutions of complex engineering problems.

Keywords: Numerical modelling, simulation, concrete, analysis, grandstands, pavements.

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Chapter 1

1.0 INTRODUCTION.

Computer simulation of structures is the discipline of developing a *model* of an actual, or sometimes physical and even theoretical, system on a digital computer, solving the model by means of numerical analysis techniques, and studying the output. If the model is successful, then the results obtained from such an analysis should be representative of the real structure itself. Therefore in theory, computer simulation can replace tasks from the real world. There is, however, a serious drawback. The reliability of the results obtained is greatly and directly related to the rigorousness and accuracy of the model developed, the available computer resources and our ability to interpret the results. Take for instance the Large Hadron Collider. Computer simulation of the collision and disintegration of the hadron particles has already been achieved and energy has been accrued. However, a 27 km underground ring site running between Switzerland and France has been built specifically, to recreate the conditions occurring at the origin of matter (Lefèvre, 2009).

Simulation is achieved mainly by utilising the Finite Element Method of Analysis (FEA) although a variety of other methods similar to the above are available today. FEA is applied to many types of problems met in a variety of disciplines. By far the most popular application is structural FEA, a revolutionary analytical tool, determining how a solid body responds to various actions. Essentially, the structural problem amounts to writing down a series of *governing equations* that describe the material and how it behaves, then solving those equations for the physical part being analyzed, subject to how it is constrained and loaded. The resulting solution provides the answer in terms of the basic variables, such as the part's changes in dimensions.

FE simulation (modelling) overlaps mathematics, physics, engineering, and computer science and is often essential when: a) the model is very complex with many variables and interacting components; b) the underlying variables are related in a nonlinear manner; c) the model contains random variants; d) the model output has to be visualised as in a 3D computer animation, for reliability reasons.

The author has been teaching numerical analysis and design techniques at both, undergraduate and postgraduate levels for over 18 years. He has undertaken extensive applied research and published mainly in the area of Concrete Pavement Structures. He has made a noteworthy contribution to the disclosure of the Falling Weight Deflectometer (FWD) as a method for assessing rigid pavement structures and has helped to make British companies aware of its potential and limitations (Hughes & Karadelis, 1998*)*. Moreover, he has made a further contribution to the subject of Non-Destructive Testing and has helped to improve the quality of roads for the benefit of road users (Karadelis, 2000). He has established an on-going research on a similar topic and is currently supervising four research students funded by Industry and EPSRC.

In parallel, the author has worked in the field of grandstand vibrations. He has developed a series of numerical models accurate enough to assess the structural behaviour of hybrid grandstands (steel skeleton with concrete terraces) under the action of human movements and predict their static and dynamic properties. He has published his findings in refereed journals; his latest publication deals with the problem of evaluating the dynamic properties of these structures (Karadelis, 2010).

The author's research concentrates mainly on the numerical analysis of structures where by employing advance computational methods and techniques he has been able to simulate reinforced concrete behaviour to solve complex engineering problems. More sophisticated models, capable of producing more accurate predictions were developed by the author as new, more accurate and reliable theories became available and significant improvements in numerical mathematics took place. The aim of these numerical models is to diminish the need and dependence on costly experimental work and replace the latter with computer representations. A few of the studies incorporating these models were published with other researchers, but most he published on his own, from 1992 to date. The establishment of Applied Research Centres and Groups (ARC & ARG) at Coventry, found the author leading an active ARG on Vibrations, Bonding and Numerical Techniques (VB&NT). He has performed well as Head of the Group and has been able to derive a great deal of knowledge from his own research and transfer it to his students. Recently, the author won an EPSRC award for more research in the area of pavement structures. Although this is still at an early stage, it is anticipated that the contribution to knowledge and the practical value of the above will be significant.

The author has made a selection of six representative pieces of his work as his main portfolio of evidence, in the form of six refereed articles as listed below. The key-words connecting all six publications are: *numerical modelling*, *computer simulation, computational techniques*.

1. Karadelis, J.N. (2000) „A Numerical Model for the Computation of Concrete Pavement Moduli: A Non-destructive Testing and Assessment Method'. *NDT&E International.* 33 (2), 77-84.

2. Karadelis, J.N., Omair M. (2001) „Elasto-Plastic FE-Analysis with Large Deformation effects of a T-end Plate Connection to Square Hollow Section' *Finite Elements in Analysis and Design*. 38 (1), 65-77.

3. Karadelis, J.N. (2009) „Concrete Grandstands. Part I: Experimental Investigation', *Proceedings of the Institution of Civil Engineers: Engineering and Computational Mechanics*, 162 (1), 3-9.

4. Karadelis, J.N. (2009) „Concrete Grandstands. Part II: Numerical Modelling.' *Proceedings of the Institution of Civil Engineers: Engineering and Computational Mechanics* 162 (1) 11-21.

5. Karadelis, J.N. (2008) „Grandstand Terraces Experimental and Computational Modal Analysis.' Schrefler, B.A. and Perego, U. (eds.) *8th World Congress on Computational Mechanics, WCCM08 - 5th European Congress on Computational Methods in Applied Sciences, ECCOMAS 2008*: *Joint IACM – IUTAM Minisymposium (MS118) on Multi-phase and Multi-scale Modelling of Concrete and Concrete Structures*. Held 30 June - 4 July 2008 at Venice, Italy. Barcelona: CIMNE.

6. Karadelis, J.N. (2010) „Reliability Pointers for Modal Parameter Identification of Precast Concrete Terraces. *Computers and Concrete, An International journal* (under review)

Four additional articles all centred on *numerical modelling* and *simulation,* are listed below as „ancillary' work. More published, similar type of work is reported in the author's curriculum vitae, enclosed.

7. Karadelis, J.N. (1998) „A Stimulating Approach to Teaching, Learning and Assessing Finite Element Methods: A case study.' *European Journal for Engineering Education* 23 (1), 91-103.

8. Hughes, B.P., Karadelis, J.N. (1998) „Computation of Rigid Pavement Stiffnesses Using Surface Deflections From The Falling Weight Deflectometer.' In *Concrete Communication Conference '98: The 8th BCA Annual Conference on Higher Education and the Concrete Industry*. Held 9-10 July 1998, Southampton University, UK. Crowthorne: British Cement Association, 87-101

9. Karadelis, J.N., Saidani, M., Omair, M. (1999) The Behaviour of T-End Plate Connection to RHS: Part II: A Numerical Model.' In Chan, S.L. and Teng, J.G. (eds.) *Advances in Steel Structures*: *Proceedings, International Conference in Advances in Steel Structures.* Held 15-17 December 1999 at Hong Kong, China. Oxford: Elsevier, (1), 313-322.

10. Karadelis, J.N. (2005) 'Strength and Failure Studies of Precast Concrete Grandstands'. In *e-Proceedings.* A*NSYS-UK European Conference 2005.* Held November 2005, Birmingham, UK. [online] available from <http://www-milton.ansys.com/ucgb/2005/ucgb_2005_proceedings.zip

Finally, this submission cites several other publications supporting the two research areas mentioned earlier, providing further evidence of an ongoing research interest. Essentially, the theme of the research and publications constitutes the platform on which both, rehabilitation of pavement structures and vibrations of grandstands are confronted. This has involved an extended, in-depth study into their mathematical representation using state-of-the-art numerical modelling techniques in both the static and dynamic domain. More specifically, the research explores the extent to which costly and time consuming and sometimes fruitless laboratory, or even full-size in-situ investigations, can be diminished and ultimately replaced by more economical, faster and profitable computer performed virtual tests. This is an emerging, new approach called *Design of Virtual Experiments* (DoVE) in which a number of analyses are run considering a number of different variables or combinations of variables. In the present studies, state-of-the-art probabilistic methods embedded into ANSYS (2002) FE software, combined with recent advances in engineering, are utilised to study the sensitivity of the (structural) behaviour of pavement overlays, or grandstands to several variables. As a result, a high quality „product' (sustainable, long-life overlays and vibration free grandstands) can be reached at a low cost and in a short time.

1.1 Aim and Objectives

The primary theme of the research is to provide accurate solutions to complex engineering problems by employing state-of-the art computational techniques. The specific themes of the author's work are explored and analysed later but on the whole, his research and publications search, identify and emphasize the following secondary themes:

- ✓ The general convenience, accuracy, savings and quality associated with computational modelling in Engineering. This is accomplished by reducing the number of laboratory and full scale tests and replacing them with computer models and in particular with DoVE.
  This is supported mainly by publications: 2, 4, 5, 6 and 10.

- ✓ The development of a series of explicit computer models based on the latest advances in numerical methods in engineering and reinforced concrete technology.
  The use of these models to gain proficiency of the behaviour of certain structures.
  Consider also the usability and applicability of these models to other engineering problems, their usefulness and effectiveness to the engineering world.
  The gradual diminishing or replacement of costly experimental work.
  Supported mainly by publications: 1, 2, 6

- ✓ The highlighting of the advantages, drawbacks, limitations and obstacles of the models, methods, techniques, procedures and solutions employed.
  The formation of design guidelines for revising and amending relevant codes, procedures and practices.

Supported by almost all publications:

- ✓ Finally, stressing the Author's contribution to research in general, and applied computational mechanics in particular and disseminating the knowledge, experience and expertise gained.
  The latter is supported by the citations of the author's work.

The above are supplemented by additional publications the majority of which are refereed articles and in a few cases by reviews and notes.

1.2 Context

Chapter II of the Critical Overview includes a brief „historic' background about the author's own work, extenuating the specific research direction chosen. A concise, state-of-the art literature review regarding strengthening and rehabilitation of concrete pavements by means of innovative overlays, as well as an investigation into the vibrations of grandstands caused by human movements are also reported. The same chapter unveils and highlights a niche, in both topics, and stresses the usefulness of a generalised model in each case. A detailed methodology follows in Chapter III, stressing the need for rigorous modelling of structural behaviour and how the selected papers can be married together under a common „key-theme'. In Chapter IV, the author describes briefly his own research and publications in the areas of concrete pavements and grandstand vibrations gradually „filling-in' the niches mentioned earlier. He discloses his own contribution mainly by reporting on the rationale, impact and value of his work and by evaluating any new knowledge offered. This leads to Chapter V entitled Synthesis; a very important chapter because it endeavours to substantiate all contributions mentioned earlier and highlights their practical importance, reliability value and effect as an ongoing process. Also, it responds to questions such as: What is the author's contribution to the particular discipline/subject and what is the new knowledge attained. Chapter VI states the conclusions and Chapter VII portrays the Future of the author's research.

Chapter 2

2.0 AREAS OF INTEREST, JUSTIFICATION AND BRIEF REVIEW.

As stated in the Introduction, the author's research interests focus in two distinct areas: Concrete pavement structures and vibrations of grandstands.

The first originated from his time at The University of Birmingham where he carried out related research, investigating the Falling Weight Deflectometer (FWD). This involved the development of a non-destructive testing (NDT) method for assessing the structural integrity of concrete pavements and predicting their remaining life. The research was sponsored by British Airports Authority (BAA) and Gatwick Airport. Literature related to numerical methods was very limited at the time. The Finite Element Method of Analysis (FEA) was not widely known, it was looked with suspicion and disbelief and, with very few notable exceptions, not included in the universities' curricula. The author was privileged to have had previous limited contact with the development of FEA during his first degree studies. Soon after he joined Birmingham he realised the powerful potential of the method, the possibility of unravelling unresolved engineering „mysteries' and the suitability of the method to become a valuable tool for his research. While FEA researchers at the Universities of Swansea and Stanford were busy developing, expanding and improving the method, relatively few researchers noticed its powerful applicability, adaptability and future potential and even they used it timidly as an „add-on' to experimental modelling for solving engineering problems. That is, the author was able to witness a niche at an early stage, from where he was to operate.

The completion of his research studies found the author spending three years in industry. However, he was soon to return to academia and research by joining Coventry University. The time was opportune to exploit existing wisdom and transform a mortar, developed mainly for structural concrete repairs (Hughes & Lubis, 1996) into a high quality concrete suitable for repairing long stretches of concrete roads and airport pavements. Hence, a new and broad research topic was born. The chief aim of the *Sustainable Concrete Pavement Overlays* is to develop a high quality, polymer modified concrete with good mechanical properties, good workability and bonding strength. In an effort to reduce costs this concrete mix should be applicable to existing worn-out concrete roads and airfields by means of an asphalt mixer and a vibrating roller. To lower costs even further, an alternative to steel reinforcement should be sought. That is, to produce a material of extensive use with the superior mechanical properties of concrete and the flexibility of asphalt.

In parallel, long established connections with the construction industry brought the author across a different type of problem. A number of sports stadia were built during the last few years following demand for multi-functional use and aiming to provide fans with more user-friendly, clear, un-obstructive views. Hence, they incorporated hybrid construction methods (steel skeleton, concrete terraces), long cantilevering segments, slender and elegant sections. These structures became very receptive (tuneful) to vibrations caused by their own occupants initiating discomfort, panic and sometimes structural damage and leading to a series of structural problems which conventional design did not cater for. Hence, a different research area emerged, that of *Vibrations of Grandstands due to Human Activities,* aiming to contribute to the study of their vibrational behaviour and ultimately rectification of the problem. Pavement overlays and vibrations of grandstands have one theme in common, in that the solutions proposed were provided by means of rigorous numerical modelling.

In comparison, few researchers have dealt with the full potential and applicability of the method, its usefulness, value and suitability within the various disciplines. Fewer have used it to the limit and reported on its performance and limitations. Therefore the author considers himself privileged to have spotted this narrow niche within the discipline of computational mechanics and applied it to two distinctive areas of his research as outlined below:

2.1 Sustainable Concrete Pavement Overlays.

It has been pointed out that better utilisation of a damaged pavement with a „white top' concrete overlay can give both environmental and economic benefits, (Britpave, 2003 & 2009). Similar research carried out to-date by the author and by others (Karadelis & Koutselas, 2003), showed the benefits of a system of suitably designed, reinforced, polymer modified concrete overlays, offering multiple crack control, high tensile, shear and flexural strengths and toughness to resist and control reflective cracking. The above, combined with good bonding qualities can be shown to satisfy all the structural requirements for a sustainable, cost effective and durable, bonded concrete overlay (Hughes, 2003).

The main aim of this research topic is to show that bonded and suitably reinforced, polymer enriched concrete overlays, constructed by utilising asphaltic paving techniques and equipment, is a very viable option. They can demonstrate unique qualities that make maximum use of the existing investment because they take advantage of the residual strength of the old pavement and combine low cost and long term sustainable solutions with concern for the environment, as they avoid wholesale replacement of the damaged pavement. It is obvious that an experimental investigation alone would involve a great deal of human, financial and technical resources and it would take a very long time to complete. Hence, the author's idea to proceed by means of computational methods and techniques and use carefully planned laboratory and site tests for the calibration and validation of the FE models only, could be an innovation of considerable value.

2.2 Grandstand Vibrations.

Recent advances in the form of construction of hybrid types of grandstands, led to longer spans, slender beams and flat, column-less slab areas. These, combined with the multi functional use of football and rugby stadia and concert halls, led to cases of these structures generating dynamic loads significantly greater than those considered in static design and therefore induce motions severe enough to cause discomfort, panic and fears about safety. The Joint Working Group (JWG) of the Institution of Structural Engineers was established in January 2000 to consider, advise and recommend on the dynamic performance and design of stadia structures. During its existence, it published an *Interim Guidance* (2001) on the dynamic performance requirements for permanent grandstands subject to crowd action, an *Advisory Note* (2002) on the dynamic testing of grandstands and a *Note* (2003) *on the calculation of Natural Frequencies* of such structures. It was actively encouraging and promoting additional research on: (a) Full scale experimentation and long term monitoring of the grandstands, (b) the derivation of the types of loads and loading scenarios generated by lively crowds, (c) the relatively unknown and complex area of human/structure interaction and (d) the accurate computer representation (modelling) of the above. It was envisaged that the above should lead to a better understanding of the behaviour of grandstands under crowd actions and make practical recommendations for design improvements.

The importance of accurately determining the dynamic properties of these structures was highlighted by suggesting that current approaches can be in serious error especially when considering continuous floors/beams, as the vibration nodes may not be at the supports (Kasperski, 2002). Kasperski added that the dynamic loading induced from an active audience becomes decisive for the design of grandstands or similar structures and that dynamic responses due to resonance with the first and second harmonics of the load may lead to considerable safety problems. He emphasised the importance of a „Monte Carlo' simulation of such loads as a significant improvement to the usual Fourier series.

Current research deals mainly with investigating the true types of loads generated by humans and their response on the structure (human-structure interaction). It has become obvious that not enough work has been directed towards accurate computer representation (modelling) of the structure itself which is crucial if a realistic approach to the design of these structures is to follow. This should act as an ancillary to current research concerning the true types of loads on these structures; hence the birth of this research project.

It has been demonstrated that there is a narrow niche in the discipline of computational mechanics which is relatively under-represented. This can be identified as the accurate numerical representation of structures in place of inconvenient real laboratory and full size tests. Structural computer representation (structural modelling) of grandstands should help to eradicate the problem of structural vibrations due to crowd movements, whereas the computer simulation of concrete pavements should provide local authorities and road users with a sustainable and financially viable solution to the problem of rehabilitation of warn out concrete roads.

Chapter 3

3.0 METHODOLOGY. COMMON THEMES OF THE SIX ARTICLES

While experimental techniques have been applied successfully for a very long time, the usage of numerical simulation is relatively low. One of the reasons for the above is the inability of current relevant software to depict real and complex engineering and physical phenomena and the resulting mistrust, disappointment and „fear of the worst', among the engineering community. The focal point of this report is the successful solution of complex structural engineering problems and in particular reinforced concrete, by using advanced and innovative computer simulations and numerical techniques. It aims at the gradual reduction of the use of laboratories and the set-up of benchmarks for computational work in the future. It also aims to help establish computational mechanics as the method of addressing and solving problems in Civil Engineering.

In order to meet the objectives set earlier and demonstrate the validity and usefulness of these techniques, two main areas of research outlined in Chapter 2 have been identified as „special' case studies. These studies are associated with a series of successful numerical models which have been developed specifically to seek practical solutions to known engineering problems in the area of reinforced concrete. In one case the theme has been taken from a slightly different, nevertheless popular area of structural engineering. Reinforced concrete, as a composite structural material, dominates both research areas mentioned earlier. Hence, it was felt appropriate to incorporate article 2, dealing with the modelling of a steel structure and therefore demonstrating the flexibility of the techniques used and the diversity and validity of the computer models.

The author himself is fully aware of the effect of non-linear analysis on the finite element method and was not prepared to accept the analysis process itself and its results at face value. Rather, he was seeking some assurance, not only that the results were up to standard but also that a sound procedure was followed in developing the models and documenting the numerous physical and numerical parameters required for a successful solution. In general, the procedures of verification and validation portray how evidence is collected and documented, to help establish confidence in the results of complex numerical simulations. In the author's research validation was used mainly for two reasons: First, to update the models by means of a manual iterative procedure, second to provide a meaningful error indicator by means of sensitivity analyses. At present, these two methods (gradual updating and sensitivity checks) work better when a sufficiently large amount of data is available. However, the author's objective is to show that even a modest amount of data is sufficient, if the finite element model is reliable.

Although the author has always tried to familiarise himself with the current literature and trends, the majority of the literature studied provided rather general benefits such as acquainting him with the discipline of numerical mathematics. In some cases this literature was very specific and narrow, as in the case of Crisfields arc-length method of avoiding singularity problems (Crisfield, 1981). Thus, the author has researched and published in relatively novel and intact, and even „unpopular' areas of engineering (reinforced concrete) and was not so fortunate to acquire a great deal of help. Nevertheless, he was able to develop the aptitude to simulate and interpret engineering problems successfully through the finite element method. As a consequence, the publications listed in the portfolio of evidence make use of the more generally established current research literature. Most empirical or practical input has been deliberately avoided in an effort to emphasise the numerical modelling techniques and link them to experimental investigations.

3.1 Linking Six-Articles Together (the common approach)

Currently, plain concrete modelling is addressed in four different domains (levels): Macro, Meso, Micro, and Nano scale levels are defined mainly via the specific interest of results expected, the amount, quality and detail of input entered and the degree of accuracy required. For instance, models at macro-scale level are assumed to consist mainly of mortar, large pores and large grains (aggregate) and necessitate the input of an appropriate level of properties. On the opposite end, models at nano-scale level are based on the hydration products as well as the un-hydrated residual clinker (cement chemistry approach) and require highly detailed and defined properties. Each scale level serves a particular purpose.

In order to capture the complex mechanical behaviour of reinforced concrete, two principal approaches were considered by the author: Meso-scale models have well defined mechanical parameters but their efficiency is very low because of the high number of degrees of freedom. Macro-scale models can be more efficient but their mechanical parameters are not so rigorous, or carefully and fully defined, and (sometimes) even undefined.

A compromise has been pursued by the author, seeking to combine the advantages of both methods with the minimum possible loss of rigor, by explicitly embedding the benefits of a meso-scopic structure into a macro-scopic model. A numerical *meso/macro-scale* approach needs the relatively complex 3-D geometry of the structure and therefore computational time and power might not be negligible. However, the benefits outperform the drawback of time. The result is a powerful numerical tool which can reveal the localised stress conditions within the structure and at the same time, resolve processes on different time and domain scales. This can be an innovation in computer simulation of structures and is adopted by the author to model reinforced concrete as a complex material.

Different 3D mechanical models were developed by the author within each domain-scale in order to depict the constitutive behaviour of the material in that domain. Each domain was given different material properties, hence characterising the structure defined within the particular domain with sufficient accuracy. These models are referred to as *Representative Volume Elements* (RVE) (Zohdi & Wriggers, 2005), (Wriggers & Hain, 2007). These virtual structures were subjected to specific loading scenarios leading to material and/or structure response. Based on the results obtained from this response, and to capture the complex mechanical behaviour, a *numerical homogenization* process was initiated (Grondin, et al. 2004) to link the discrete meso-scopic structure to the extended (macro-scale) continuum formulation. Hence, the resulting constitutive equations were applied to the next domain-scale by the program to model the constituents belonging to that scale. This way a representative, realistic and wider material description was achieved from the beginning to the end of simulation.

Subsequently, Papers I and II deal with materials and geometry representation in the non-linear domain. For the pavement structure, a non-linear finite element model was developed based on the general representation of linear elastic - strain hardening (partly plastic) behaviour of soils and their inability to resist tension. In the case of steel connection the model adopted was based on the linear elastic- (near) perfectly plastic approach (bi-linear representation) which fits steel behaviour more realistically. The same approach was used to model the steel reinforcement in reinforced concrete. In both cases the modified Newton-Rapson method for the solution of the equations was used as no profound singularity problems were noticed. Hence, the simulation process was similar although the two materials and their performance in general are different.

Papers III and IV focus on concrete uncertainties and non-linearities and how the latter could be represented in a realistic numerical model. Initially, they built upon a general elasto-plastic approach and at a later stage they were introduced with cracking and crushing failure characteristics. The author referred to the strain softening effects and pointed out that the well-known cube/cylinder compression tests may not be representative to describe the stress-strain performance of concrete. He also referred to the solution procedure and commented upon the suitability of Crisfield's method to represent strain softening effects. Papers V and VI commence the dynamic analysis of concrete grandstand terraces attracting knowledge from the statics models. The results obtained were found to be highly accurate but also very sensitive to the correct representation of boundary conditions. This also sustained an earlier argument that the methodology followed to simulate concrete and concrete structures at meso/macro scale level can produce good results.

It is therefore clear from the above that the dominating theme of the selected six papers is the numerical modelling of reinforced concrete in the non-linear, more pragmatic domain. A common purpose could also be identified, that is, to study the behaviour of certain structures under different loading regimes and predict conditions otherwise not easily seen, detected or understood. This way, structures can be optimised for efficient and effective use. Essentially, the reason for research into grandstand vibrations is to improve comfort and safety and reliability and amend and update current codes of practice. The main purpose for concrete pavement overlays research is to reduce the number of accidents on roads, improve riding quality and offer environmentally friendly and sustainable solutions to tight infrastructure budgets.

Finite element analysis is based on modelling engineering behaviour rather than geometry of structures but in many cases this may not be as obvious, or simple. This has an effect on the particular underlying principle (logic) followed which may have common characteristics in many structures. For instance, all the selected papers have their geometry developed to a certain level, with details not considered to be of structural importance left out. Material properties were always pre-estimated, usually by standard laboratory means and then inserted to represent the model. Loading systems were prescribed in an incremental manner (load steps) for better convergence control but also to be able to obtain output to specific load stages.

Prior to the analysis run, some important „control' parameters such as iteration frequency, convergence criteria, load increment and bisection control, and stiffness update strategy were selected as input to define and adjust the algorithm during the solution process. The stiffness update frequency and contingencies for divergence and „oscillations' were also allowed for. The code permitted some of these parameters to be automatically updated during the solution process, to efficiently respond to the nonlinear environment. These parameters were important because they helped to overcome numerical difficulties encountered with the mixing of different elements, hence they were used frequently.

Chapter 4

4.0 AUTHOR'S OWN RESEARCH IN THE AREAS OF CONCRETE PAVEMENTS AND VIBRATIONS OF GRANDSTANDS.

The constitutive relationships in structural analysis connect the applied forces or stresses to deformations or strains. In their simplest form, that is, for materials exhibiting linear elastic behaviour these relationships are also linear, following Hooke's law. This is not the case with reinforced concrete. For a deeper understanding of the constitutive behaviour of multi scale materials like concrete, one should perform a series of experimental investigations and/or numerical simulations. While experimental techniques have been applied successfully for a very long time, the successful usage of numerical simulation is very limited. The author believes that although with the currently available software and hardware it is certainly not practical, if not impossible, to attempt to model concrete in all four levels (nano/micro/meso/macro scales) at the same time, it should however be possible with the aid of today's knowledge and technology to develop rigorous models based on combinations of the above and obtain reasonably accurate results.

A numerical multi-scale simulation approach may require as input, among other parameters, the complex three dimensional modelling of the structure of the material at atomic, molecular, constituent, elemental and sub-structuring levels and therefore colossal computational power, effort, man-hours and expertise, resources which only a handful of organisations on earth can demonstrate today.

The aforementioned numerical multi-scale simulation approach could, in theory, be suited to concrete, itself being a complex material. However, as this is currently impractical, a similar, compromised approach of investigating concrete should give sufficiently accurate results in most cases and at the same time point towards future improvements. The author's recent work is based on the above hypothesis. The author has identified a niche within the vast computational analysis world and has concerted his efforts in filling this gap with his own work. A short but critical account of the author's own research work is presented below.

4.1 No. I, Journal Paper

A Numerical Model for the Computation of Concrete Pavement Moduli. –A Non-destructive testing and assessment method.

4.1.1 Rationale

The paper describes the development of a non-destructive method, integrated within a pavement management system capable of extracting information about the structural condition (evaluation) of rigid pavements by utilising the Falling Weight Deflectometer (FWD) (Karadelis, 1990), a machine which had previously been used reasonably effectively for flexible pavements.

A systematic search on both rigid and flexible pavements in general, and the FWD in particular, provided the author with the ideal background and basis to inspire the development of the method. A series of site trials with a FWD machine had been crucial in obtaining confidence and practical experience and validate the method at a later stage. In addition, a series of laboratory tests had been central to complement the site trials.
All trials were carried out within designated sites at Gatwick airport using a hired FWD machine. For the in-house tests a „static' version of the FWD was designed and built in the laboratory by the author. It was capable of producing a scaled down impact load on a circular concrete slab resting on a deep layer of sand. Surface displacements and strains were measured at five positions along a radius. Stresses at various depths within the sand were also recorded. At the same time, two non-linear finite element models were developed by the author, retrofitted with information obtained from the laboratory and the Gatwick site respectively. The models were capable of matching deflections obtained in the laboratory and on site and used to predict corresponding layer moduli.

It was envisaged that if the surface deflections, stresses and strains within a pavement system were measured simultaneously under a given load and then correlated with theoretical values obtained from a finite element analysis of a similar pavement model, then the assumed theoretical moduli should be representative of the particular pavement materials.

4.1.2 Contributions

The resulting FE model was capable of depicting all behavioural patterns of a real pavement, except one. Wheel load transfer between two adjacent pavements (e.g. across joints) takes place by means of shear (i.e. dowel) action bars connecting the pavements. When modelling a pavement in 3D it is not difficult to insert BEAM elements representing the dowels, attached to SOLID elements representing the concrete material. However, in a 2D, axi-symmetric representation it would be erroneous to model load transfer in a discrete manner because the idea of „solid by revolution' would be violated. Two dimensional simulation has several distinct advantages over the 3D but it also has a drawback. It cannot model conditions of this kind as dowel bars are not „continuous' in a cyclic manner.
The problem of load transfer between pavements was therefore addressed by utilising the concept of mechanical efficiency, such as:

$$n = \left[\frac{\delta(u)}{\delta(l)} \right] \times 100 \qquad (1)$$

where:

n= efficiency as a percentage

$\delta(u)$= displacement of unloaded slab at a joint

$\delta(l)$= displacement of loaded slab at the same joint

Typical values of efficiency of approximately 80% were introduced to the model in the form of prescribed displacements. A 2-Dimentional, 4-noded isoparametric (the same shape function is used to generate the stiffness and mass matrices), quadrilateral, plain stress finite element, with 2 DOF (degrees of freedom) per node and axi-symmetric capabilities was used in the analysis, as opposed to a 3D solid/prismatic one, reducing the analysis time and effort substantially and without compromising the results. It is shown in Figure 1.

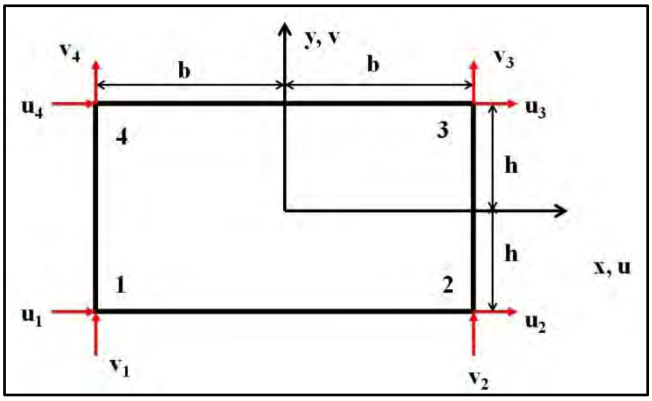

Figure 1. A 2D, 4-noded, isoparametric rectangular element with 2 DOF per node.

Linear elastic analysis cannot depict the structural behaviour of a material like reinforced concrete and of course, soil and therefore deviation to non-linear representation was a necessity. However, it was not practical to „non-linearise' all 5 layers of the Gatwick Airport pavement model, mainly due to restrictions in computer power and CPU (Central Processing Unit)-time. While seeking a compromise, a carefully conducted sensitivity study showed that the soil layer had the most profound effect on the results. Therefore, it was decided to address the soil non-linearly. A cubic spline function was developed to interpolate between a set of experimental stress-strain data. Some of these data were the result of routine, laboratory based, triaxial tests carried out on soil samples taken from the designated site. Some were also collected from the purposely instrumented pavement site. These data, representing soil material properties, were fed directly into the system in the form of stress-strain points. The program treated these points as tangent moduli on the theoretical (spline modelled) stress-strain path. The tangent moduli were capable of producing stress and strain results similar to those measured earlier, and therefore it was concluded that the inserted tangent moduli were representative of the pavement system. Finally, these moduli were related to surface deflections using the theory of load distribution beneath a loading plate approximating a truncated cone (Lytton & Smith, 1985). A purely exponential function was also introduced to represent an all-purpose soil behaviour when no such data were available. The exponential function developed for this purpose, was in this case tailored to model Gatwick soil.

In addition to the complex structural modelling and the engineering logic developed specifically for the research, the author made a significant contribution to the disclosure of the FWD as a method for assessing rigid pavement structures. He assisted in making Local Authorities and British companies aware of its potential and limitations. Moreover, he made an added contribution to the area of Non-Destructive Testing and helped to improve the quality of roads for the benefit of road users. Finally, British Airports Authority (BAA) and Gatwick Airport were the direct beneficiaries as they sponsored the research.

4.1.3 Benefits

The use of symmetry is strongly encouraged in finite element procedures as it dramatically reduces CPU (central processing unit) time, makes FE-running possible on low specification PC platforms and eases significantly the interpretation of results. At the time, finite element programs were running on mainframes only. Available memory was critically limited as also was the maximum number of elements and nodes permitted. As the focal point in this case was not to study the behaviour of a particular pavement system, but to test the suitability of the FWD and develop a method to evaluate the structural condition of rigid pavements, it was decided that an axi-symmetric, two dimensional (2D) representation of an otherwise solid, three dimensional (3D) prismatic structure should suffice. This produced the same quality of results as a 3D analysis but with only a fraction of the effort, time and cost.

The development and use of a cubic spline and an exponential function solved the problem of complex, non-linear soil representation. The result was a twin numerical model capable of addressing more than one problem and therefore demonstrating flexibility, effectiveness and efficiency. It was envisaged at the time, that with the advent of more powerful generations of personal computers this representation could be extended to concrete. This materialized a few years later (Paper IV). When information of the soil conditions was available, the cubic spline was utilised, closely predicting a non-linear performance of the pavement system. On the other hand, when no data existed, the performance of the pavement system was predicted by the exponential function which was chosen to represent more general soil conditions of the site in question.

4.1.4 Originality

At the time the Falling Weight Deflectometer (FWD) was only used to evaluate flexible pavements because the impact load it produced was not capable of causing a measurable deflection to a material with the hardness and stiffness of concrete. The author experimented with a "heavy" version of the FWD, which was capable of producing realistic deflections. The deflection „bowl' formed under the falling mass was different in nature to that obtained from a flexible pavement. Hence, the mathematical approach as well as the evaluation methodologies developed were also different but nevertheless sound and valid and therefore his work was considered to be original. The methodology describing the use of the cubic spline function and its inter-changeability with an exponential function was unique. The former had some limited use in Aeronautics and Naval Architecture research only. Finally, the decision of the author to use „indirect' methods to model engineering phenomena (efficiency and shear transfer) was an additional innovation in engineering simulation.

4.1.5 Value, Impact, Citations

A serious drawback of any axi-symmetric simulation is the inability of the technique to represent accurately all non-symmetric problems. The introduction of mechanical efficiency, as described previously, was of particular importance, as it simplified and depicted a strictly non-symmetric problem in the axi-symmetric solution domain. This had a significant impact in FE analysis as effort and time were dramatically reduced and the load transfer problem was addressed and solved in a simple, accurate and effective way. The two-way computer representation of the soil layer (third degree spline utilising some experimental data, or an exponential function when no data were available) as part of a pavement structure has made a significant difference in the assessment and evaluation of concrete pavements. Airport and transportation engineers should now be able to take advantage by assessing pavements in a fast, economical and effective way. Finally, it was demonstrated that in analysing pavement structures there was no real need for a thorough non-linear representation of all layers, hence reducing the time and effort of the analysis itself.

The author's work has made a particularly good impact among fellow researchers. First, Matsumoto (1995) referred to his work in a state-of-the-art report on concrete research in UK. Grenier & Konrad (2002) cited the author's work and used his findings to calibrate their FWD to detect cracks in asphalt pavements and study the influence of temperature changes on the deflection „bowl' formed under the falling weight. Zhi, et al. (2004) used the author's FE model as a basis to develop their own. They noted that FE models cannot describe, mathematically or otherwise, the complex profile of the road surface and concluded that adapting to the shape of any road surface is rather a random process. LV Hui-qing, et al. (2005) presented a study of the mechanical properties of cement concrete pavements and cited the author's work in their effort to relate the limitations and merits of the various models with their capabilities. Koushki, et al. (2006) referred to the same paper in assessing the compatibility between destructive and non-destructive tests of concrete strength. Also, Xiang & Chen (2006) utilised the author's article by adopting his axi-symmetric numerical models. They used these models to analyse the dynamic effects of impulsive loads produced by the FWD on the stiff layers. Similarly, Antonaci, et al. (2007) reported repeatedly on the author's exclusive methodology in extracting moduli values (and therefore the structural condition) from a rigid pavement system. Saltan, et al. (2008) cited the author's work by referring to, and implementing his version of FWD. Very recently, Saltan & Terzi (2009) used the author's methodology as portrayed in his article, to develop an adaptable neuro-fussy inference system for the back-calculation of pavement layer moduli and thicknesses in a multi-layer pavement system. At about the same time Park, et al. (2009) followed a similar procedure. After applying successfully a genetic algorithm and the finite element method they too were able to back-calculate layer moduli, this time for flexible pavements. The researchers praised the author's well-established work on the structural evaluation of concrete pavements. Finally, Saltar, et al. (2010) used the author's methodology and employed data mining (DM)-based pavement back-calculation tools for determining the *in situ* elastic moduli and Poisson's ratio of asphalt pavements from derived Falling Weight Deflectometer (FWD) deflections at seven equidistant points.

4.2 No. II, Journal Paper

Elasto-Plastic FE-Analysis with Large Deformation effects of a T-end Plate Connection to Square Hollow Section.

4.2.1 Rationale

Paper No. II deals with the performance of a popular type of structural steel SHS (square hollow section) connection usually met in roof truss construction. Its research aimed to investigate and optimise the structural connection by integrating two distinct types of analyses. Initially, the system of SHS, cap-plate, cleat-plate and the surrounding weld was represented as made of one material with (classical) bi-linear material properties. The system was placed under axial tension and an analysis was performed producing acceptable results. In an effort to improve results, the model was revised and the weld was given different, more realistic properties as advised by the manufacturer. It was also recognised that the analysis as stated above, did not depict possible out–of–plane large deformation (buckling) effects of the cap-plate. Therefore, large displacement analysis effects were introduced in addition to the existing material non-linearities for the latter component. It was noticed that a better agreement was achieved between experimental and theoretical results. The latter helped in optimising the design of the cap plate further, by taking advantage of this interesting stability phenomenon where a structure "snaps through" from a stable formation to an unstable (snap-through) to another stable condition.

4.2.2 Contributions

This work produced useful guidelines for the optimum design of a popular family of structural connections, hence making a contribution to all those involved in their design. Often, it may be difficult to detect and recognise problems involving large deformation effects and allow for these in a non-linear representation. It is therefore advisable to have the option open during the analysis process. This will not affect the quality of the results if large deformations do not, finally, take place. The option is highly recommended by the author when similar work (analysis of plates) takes place and especially when design optimisation procedures are sought. It should also be very useful for revising existing empirical design procedures. The methodology followed and the savings achieved by allowing the plate to undergo large deformations should be of particular interest to steelwork designers and constructors.

4.2.3 Benefits

The benefits obtained from a combined material non-linearities and large deformation analysis are not always obvious. In certain slender, or flat types of structural components (pipes, plates, panels, slabs) membrane stresses may cause the structure to stiffen and hence reduce displacements. The analysis described above used this phenomenon to its advantage. Admittedly, the latter are special effects, not always present. However, if the structure undergoes large displacements or deformations and this is not depicted in the solution procedure, then the best possible solution may not be achieved. This type of analysis may be applicable to a variety of engineering problems, especially those needing optimisation procedures.

4.2.4 Originality

The idea of representing steel properties in a bi-linear manner is not original. However, the combination of material and geometric non-linearities has not been popular among researchers and practitioners. This article was published in a well known international journal, because of its clarity in addressing non-linear matters and its good quality and accuracy of results. It deals with a popular structural steel connection that can be used as a benchmark for other similar types of structural connections. It has also shown the way to researchers working on similar areas that it is possible to achieve the most favourable results by optimising the performance of each individual component rather than considering the structural connection as a whole.

4.2.5 Value, Impact (difference), Citations

It has been substantiated that it may often be worthwhile to include the option of large deflections open, as it can notably improve the quality of results. Without the above option correlation between experimental and theoretical results would be limited and the whole analysis procedure, although in the non-linear domain, would be incapable of depicting the true performance of the structural connection in the laboratory. The article has appeared in the archives of the American Academy of Mechanics (2001) and has been cited by Li and Yin (2001) in their study of the behaviour of bolted connections at elevated temperatures. Also, it was quoted by Li & Li (2006, 2007, 2008) in three successive publications. In one of their papers, the researchers refer to the author's exclusive idea of large deformation effects at three different locations in order to develop a „dual function' metallic damper with enhanced stiffness and seismic energy dissipation capabilities. In a second paper, a modified version of the author's numerical model was also used by the same researchers. Finally, Kiral & Erim (2006) used information and findings from Paper II, to examine the performance of steel-to-column connections with weld defects under hysteresis types of loads.

4.2.6 Statement of Collaboration

All appropriate laboratory work as referred to Journal Paper II has been conducted by Dr. M Omair, to whom the author is grateful. Dr Omair was a research student of the University at the time. The author was asked to advise him on the theoretical side of his work, resulting in this publication.

4.3 No. III, Journal Paper

Concrete Grandstands. Part I. Experimental Investigation.

4.3.1 Rationale

This paper describes the experimental investigation of a set of three identical precast concrete terrace units and their behaviour under static loading-unloading. Three main objectives were set: First, to examine the structural behaviour of a family of RC terrace units supported at three predefined positions and undergoing static loading-unloading. Second, to estimate the uncracked and fully cracked stiffness of the units and use them in a dynamic analysis. Third, to provide a firm platform on which a rigorous numerical model could be built and used for further analysis work. The initial design of the units was carried out by BISON Concrete Products Ltd. It was evident that for simplicity, the units were considered supported at the two ends only and designed accordingly. However, it was also evident that this approach did not depict realistic site conditions.

Support of these units is often provided by steel raker beams on which a series of steel plates (stools) are welded to provide the end bearings. The front edge of the tread rests on the „riser' of the lower unit and so on. The raker beams were not reproduced in the laboratory due to limited space. Steel stools were inserted under the ends of the riser and a suitable UB-section was placed under the front end to replicate the line restraint by the lower unit. Loading was applied by means of 6 point loads along the tread, arranged at a distance of 100 mm from the riser in an incremented-decremented manner. The deflection at mid-span and the maximum strain in the main longitudinal and lateral reinforcement was recorded, as well as the strain at ten different positions, against the applied load.

4.3.2 Contributions

It is not considered to be standard practice to perform full size physical tests in any laboratory mainly due to the limited space available. However, the benefits of such tests are well understood and welcomed by academics and especially by practicing engineers who have in the past been slightly cautious of any new theoretical development and especially numerical analysis techniques. Tests of this kind are always regarded as an improvement and a contribution to the engineering community, as well as a valuable aid to numerical analysis work.

Specifically, the maximum displacement of the units measured at mid-span was found to be approximately equal for both uncracked and cracked units. This would demonstrate a similar behaviour before and after cracking, somehow emphasizing already established reinforced concrete section theories. Also, strain distribution at mid-span across the riser was found to be linear and similar in both cases. However, a more complex deformed shape of the units was revealed, when a ‚*trough*' developed at the central region. At the same region the tread, situated below the neutral axis of the riser, followed closely the behaviour of the latter developing tension at the top. This could have certain implications on the design of the units, as the tread did not seem to be appropriately reinforced in the locality. In fact, it is understood that considerable savings could be achieved if the units were designed using plate theory rather than being considered as simply supported beams. The above is reviewed later when a finite element model is under consideration. It is expected that the published results will be utilised by a number of researchers who would like to build upon current strengths.

4.3.3 Benefits

The use of full-size, RC laboratory investigations has historically been associated with many practical and financial problems and has generally been evaded by researchers. Hence, any type of related work can only be welcomed by all those interested. Owing to its predictable properties, it is now postulated and established that most numerical models featuring steel as the main material may need no further validation. The same cannot be said for reinforced concrete. Although its behaviour under complex stress conditions has been under investigation for years, there is as yet no universally accepted constitutive law established. Hence, any experimental „add-ons' are always welcome. This series of carefully planned and executed full-size laboratory tests proved to be invaluable for calibrating, fine-tuning and finally validating the numerical models developed later. The same tests may be useful to the wider academic and practising communities as they can provide useful information and experiences for similar types of work. Also, they have been used several times by the author as an aid to teaching reinforced concrete theory to his students of Civil and Structural Engineering.

4.3.4 Originality

The author is not aware of similar earlier attempts to assess the performance of full-size grandstand terraces in the laboratory. It is expected that this study will be very useful to precast concrete manufacturers and design consultants of grandstands and other similar structures. The results obtained are original and of good quality and portray the real structural behaviour of the terrace units, not the one assumed in their design.

4.3.5 Value, Impact

Article No. III has been published recently and therefore it is relatively early to assess its impact by the number of times it is quoted elsewhere. The author believes that other researchers may be using his findings to develop their own hypothesis and even utilise his experimental results to validate their own models. Also, the paper should be of value to precast concrete manufacturers who specialise in similar types of structures and essentially in the design and construction of grandstands. Living in a highly competitive technical world any savings in materials should be welcomed and regarded as a matter for survival. Therefore, revising their design according to the findings of this investigation should be beneficial. Finally, these tests have become the „validation certificate' for the numerical model described below.

4.4 No.IV, Journal Paper

Concrete Grandstands. Part II. Numerical Modelling.

4.4.1 Rationale

Paper IV constitutes the second part of the preliminary investigation of grandstands under dynamic actions. It should be read along with the first part which is merely an experimental investigation. A systematic approach to this otherwise dynamic research topic necessitated an initial investigation in the statics domain, in order to develop, test and validate the most appropriate numerical models which were later to be put under the scrutiny of dynamic actions. The article portrays the development of a rigorous finite element model incorporating special material non-linearities for both concrete and steel reinforcement. Failure criteria for both materials were based on their distinctive modes and mechanisms of failure. For the concrete in particular, these were yielding, load-carrying capacity, cracking and crushing and deformation. In an effort to approximate as close as possible the structural behaviour of the concrete-steel composite action the latter were input in the program by means of an array of data taken from routine laboratory tests, the corresponding experimental investigation and established theories.

Numerical weaknesses such as the representation of the problem of shear transfer from the tension reinforcement to the aggregate interlocking mechanism and finally to the compression side of the structural section were discussed and highlighted and a way out of the problem was proposed. The benefits and drawbacks of the smeared reinforcement offered by a specific element were critically reviewed. A section was dedicated to the „softening' behaviour of concrete and a critical debate was initiated as to whether the dramatic reduction of stress relative to strain after a peak, does represent realistic (brittle) concrete behaviour. Finally, the interaction between the two materials and the concept of composite action in conjunction with the development of a meso/macro-scale mechanics model like the present was considered.

4.4.2 Contributions

With the current paper the author made a number of well-founded contributions to the general structural modelling and computational mechanics world as well as to reinforced concrete as a material. He developed a most up-to-date finite element model of a reinforced concrete terrace unit with help from a parallel laboratory investigation. The author is confident that the general elasto-pastic, meso/macro scale level approach, combined with cracking and crushing modes of failure, has captured the non-linear behaviour of the unit to failure.

The author's decision to steer away of the currently unresolved issue of strain softening and introduce more realistic failure criteria for concrete proved to be successful. It is believed that if a similar approach is adopted for the bond failure between concrete and reinforcement, that is, if the process of debonding can be described in a more rigorous manner rather than as a sudden local failure, existing results should be substantially improved. In the current investigation, the problem of shear resistance of the main reinforcement was resolved by assigning the contribution of steel to the surrounding aggregate. This may not be entirely compatible with existing design models but in the absence of more realistic and accurate numerical representation it may be considered adequate and has produced acceptable results.

4.4.3 Benefits

The „sensitivity' of the non-linear model developed, depends on both cracking and crushing of concrete and the yielding of the reinforcement. This makes the model particularly attractive to practising engineers who deal with the design of „special' structures and are seeking advice and solutions beyond the conventional codes of practice. The problem of shear transfer was resolved by assigning the amount of shear resistance normally provided by the re-bars, to the interlocking ability of the surrounding aggregate. Moreover, when shear transfer (combined with tensile stresses) takes place the bond between steel and concrete starts to break down. Shear resistance is gradually passed to the interlocking ability of the coarse aggregate within the concrete itself until the re-bars can resist very little or no shear at all. Bond failure is a long process with the concrete material surrounding the reinforcement starting to slip long before the maximum bond strength is reached. Hence, although restricting bond stresses to a specific value would sound as a possible alternative, it would not however be representative. It is pointed out that the complicated bond mechanism of failure was outside the scope of a macro/meso-scale model like the present.

There were more benefits associated with the representation of shear resistance. ANSYS recommends a dedicated 3D, eight node, solid, isoparametric element (SOLID65), with three translational DOFs per node, suitable to simulate the non-linear response of brittle materials like concrete. In addition, ANSYS recognises that concrete needs reinforcement and therefore allows an additional smeared stiffness distributed through the centroid of the element in a 3D Cartesian system orientation. Thus the analyst can define up to three different size re-bars in three different directions. These virtual rebars can resist tension and compression but unlike the real reinforcement, they cannot provide any shear resistance. Neither the BEAM nor the LINK families of elements would solve the problem to its roots. The BEAM elements, although they would resist shear they are however purely linear elements, not suitable for non-linear analysis. The LINK elements on the other hand, like their smeared opposite numbers, would go beyond yielding but would not resist shear stresses. It seems therefore that there is no suitable element for RC representations.
Another drawback which is not perhaps highlighted enough in the paper is that, as the smeared stiffness is distributed through the centroid of the element in three orthogonal directions, its position and orientation is not always suitable to represent that of a typical system of steel bars (eg: tension, compression and shear reinforcement in a beam).
The author proposed a discrete representation of the reinforcement to address the problem of shear resistance by choosing LINK8 elements. This way the problems of position and orientation of the reinforcement were solved while the contribution of shear resistance was attributed to aggregate interlocking ability, as discussed earlier.

4.4.4 Originality

Paper No. IV demonstrates a number of original qualities. For example, it addresses the problem of shear transfer (other researchers have conveniently avoided it) in a simple and effective manner. The capturing of flexural cracks has opened new horizons in modelling failure conditions of reinforced concrete structures. It is now easier to predict more complex modes of failure such as the combination of flexural and shear failure which is the most popular in reinforced concrete beams. The algorithm presented has been tested and validated with experimental data and therefore it is reliable and relatively easy to adapt and tailor to particular circumstances. Hence, the model can be used as a benchmark for further numerical analysis work into the area of reinforced concrete. The phenomenon of strain softening is by no means original. However, only a few researchers in the past have raised concerns about its suitability to represent concrete (Hughes, 1976), (Kotsovos, et al. 2005). The author was able to report on both arguments and express his own reasoned opinion. Finally, the initiative to deal with these types of problems at a macro/meso scale level has worked well and may be opening new horizons to practicing engineers and analysts and in particular those who are dealing with the simulation of large, civil engineering structures.

4.4.5 Value, Impact

It has been demonstrated by the author that the dedicated concrete element in ANSYS is not always the best option in modelling reinforced concrete and its performance under loading. However, the proposed combination of SOLID65 and LINK8 elements may be adequate to depict the full performance of RC terraces or, indeed other structures undergoing incremental loading. This is expected to be of value to researchers and practitioners who are interested on a relatively simple but rigorous solution of their problem. Also, the problem of transferring shear stresses from the main reinforcement to the concrete material was addressed by the author in an indirect and simplified manner and is expected to be utilised by others. It is relatively early to report any citations but both articles have enjoyed good comments from the Chairman of the editorial panel (Bothwick, 2009).

4.5 No.V, Conference Paper

Grandstand Terraces. Experimental and Computational Modal Analysis.

4.5.1 Rationale

Paper No. V can be regarded as a milestone in the Grandstands research as it consists of the first investigation in the dynamics domain. The principal reason was to report on the performance of these large structures under dynamic actions and if possible, use the knowledge to develop a broad but rigorous global model of an entire stadia structure. Another important reason was to obtain the natural frequencies and mode shapes of the terrace units and use them as input to subsequent harmonic and random vibration analyses. The wisdom obtained by developing finite element models in statics was exploited. Data and results collected from full size modal analyses tests on single and double terrace units were processed and utilised accordingly. These were compared with the corresponding natural frequencies and mode shapes predicted by the finite element models in order to obtain proficiency and validate the theoretical model. A good correlation was achieved and a series of conclusions and recommendations were drawn.

4.5.2 Contributions

The author was able to draw a series of general but useful comments as follows: He indicated that experimental modal analysis alone may not be adequate to provide a complete and accurate description of the performance of certain structures such as grandstands undergoing complex modes of vibration. This is because some modes may be complex enough (coupled modes) not to be depicted by the available testing equipment. He pointed out that experimentation should be accompanied by finite element analysis, in an effort to understand and depict more accurately the resulting mode shapes, especially at higher modes of vibration.

There are conflicting messages in the engineering world regarding the influence of reinforcement in the dynamic performance of structures. Previous studies (Numayr. et al. 2003) and short term tests suggest that the amount of reinforcement has only very little effect on the dynamic properties of an uncracked reinforced concrete beam. However, the definition of an uncracked beam is rather conceptual as all RC beams would have hair-like cracks developing due to shrinkage. Also, no beam remains uncracked after a few load increments long before it reaches its ultimate load value.

Interim studies by Pandeli & Karadelis (2003) hinted towards the possibility that an increase in the amount of reinforcement is likely to force the structure into a different mode of vibration, hence altering the previously obtained dynamic properties. In the same studies it was shown that, introducing and gradually increasing the tension reinforcement resulted in a marginal increase of the first two natural frequencies associated with bending modes but had no effect on the next two, mainly torsional modes of vibration. A similar performance was also noted in the present study. He concluded that changes in the dynamic properties may not be detected by increasing the amount of one type of reinforcement alone, because any significant change in the dynamic response of the system is bound to take place gradually and because there is not enough evidence to compare the different states obtained. The authors concluded that more work is needed for patterns to be formulated and benchmarks to be established.

In addition, he concluded that testing or modelling parts or elements of a structure only, does not necessarily provide a global understanding of the structure's dynamic performance. Essentially, he noticed that, at certain modes of vibration, the two terrace units moved independently and the motion of one had the tendency to cancel the motion of the other. Therefore, one cannot generalise or conclude for the whole grandstand by simply studying one, or two, units alone. It has therefore been concluded that more methodical work is needed for patterns to emerge.

4.5.3 Benefits

The model developed established the basis for a further, more systematic modal parameter identification method, based on a continuously updated model that is demonstrated in the next paper. It was also shown that experimental modal analysis combined with a rigorous Finite Element approach can reduce uncertainties and produce reliable results. This has the potential to be a very accurate, generalised method of determining the dynamic properties of structures. By applying symmetric conditions the number of nodes and DOFs used in the model was reduced substantially. This has great benefits on the CPU time as well as the time and effort required by the analyst and in certain cases could even be obligatory due to limitations of the software itself.

Finally, it has been demonstrated that experimental techniques alone may not be accurate enough, and sometimes even misleading, in determining the dynamic properties of a structure. This may be particularly profound at the stage of results interpretation and therefore a combination of the latter with a finite element model can improve quality and reliability and become a valuable tool to specialist consultants, and a benchmark to academia for further research.

4.5.4 Originality

The author was able to demonstrate that in the case of relatively complex reinforced concrete structures, experimental modal analysis alone may not be adequate to provide a complete and reliable account of its dynamic behaviour unless perhaps, a great deal of experience in interpreting results combined with state-of-the-art equipment is also available. The author has recommended that laboratory or site tests are accompanied by the appropriate finite element model to provide higher accuracy and reliability, although it is accepted that it is not very often that a finite element model can be regarded by professionals as comprehensive and reliable as real, full size tests.

Related studies and short term tests and results suggest that the amount of reinforcement has only very little effect on the dynamic properties of the uncracked reinforced concrete terrace units. Yet, our own studies hinted towards the possibility that an increase in the amount of reinforcement is likely to force the structure into a different mode of vibration, hence altering the previously obtained dynamic properties. The role of reinforcement in the dynamic properties of a structure should be the basis for more research.

Finally, It has been demonstrated that the dynamic properties of a RC terrace unit are very sensitive to the type of supports provided; the latter requiring as accurate modelling as possible. As mentioned earlier, symmetry was considered in order to reduce the number of nodes and DOFs used in the model and to optimise CPU-time and computer performance. It was also stated that the application of symmetry may result in the exclusion of one or more mode shapes. It is recommended that if the analyst has reasons to believe that this is the case, the best solution may be to model the whole structure, using a coarse mesh in order to extract the mode shapes and then use symmetry and a dense mesh to obtain the natural frequencies. Nevertheless, it is likely that only higher order mode shapes and natural frequencies are lost, if any, with symmetry which in general, are of no importance to structural engineers.

4.5.5. Value, Impact

Paper No. V has been published recently and its value to the academic and practicing world is still untested. It has made a good impression and has received encouraging comments by academics and other professionals at the international conference where it was presented. It is hoped that the current article, as well as those to follow as an output of the grandstands research, will not only make a strong impression but they will also serve their purpose and make a constructive and positive contribution to this area of research.

4.6 No.VI, Journal Paper

Reliability Pointers for Modal Parameter Identification of Precast Concrete Terraces.

4.6.1 Rationale

The final paper demonstrates the author's efforts to continue with his research on grandstand vibrations. Although the subject matter is related to Paper V, the two papers are quite different. Paper V presents a general account of experimental and numerical modal analysis, as it is usually the case with conference papers, examining the trends, problems and drawbacks of modal analysis. It does not propose any tangible solutions; this is dealt with in the current paper. In contrast, Paper VI focuses on the numerical side, and how good research strategy can accomplish modal parameter identification effectively.

4.6.2 Contributions

The author involved himself in a critical discussion and comments on the different solution algorithms (eigenvalue extraction methods) supported by ANSYS and their suitability in solving problems in the dynamics domain. He went on to compare two well known solvers: The Subspace, based on a generalised Jacobi iteration algorithm (Mahinthakumar & Hoole, 1990) and the Block Lanczos, which uses the Lanczos algorithm performed with a block of vectors (Lewis, et al. 1994). He concluded that the latter performed particularly well by reducing CPU time and adding to the accuracy of the results and he recommended it for similar types of numerical work.

In addition, the author demonstrated that the built-in the program support restrains were inadequate to model complex support conditions (boundary problems). He realised that the overall performance of the grandstand units was highly reliant on the stiffness of the supporting structure and in particular on the elastomeric bearings (pads) inserted between the underside of the concrete unit and the supporting steel frame. After consulting the manufacturer's literature, the author developed and introduced a stiffness matrix featuring shear and compressive stiffness characteristics for the pads. A spectacular improvement of the quality of results was noticed immediately, that led the author to highly recommend his findings for similar work.

Significant improvement of the results was also achieved when the properties of concrete were revised and more realistic and justifiable values were introduced. In fact, accurate support representation combined with a careful revision of the material properties were responsible for an improvement of more that 60% in the model updating process and the correlation achieved. More parameters were identified as having an effect on the dynamic properties of the units and were included in the updating process.

The author concluded that experimental procedures alone may not be adequate to provide a complete account of the dynamic behaviour of grandstand terraces, as they greatly depend on high standard, expensive equipment and highly skilled operatives. He suggested that a rigorous finite element model should always be an inseparable companion to modal tests and demonstrated that in some cases (complex modes) it could even take the leading role and provide reliable, quality results.

Finally, verification of the experimental and numerical results may not always be possible, or sometimes may be impractical due to complex mathematical calculations and a plethora of uncertainty factors arising mainly from the constants involved.

4.6.3 Benefits

It was demonstrated that experimental methods and procedures may not always be the most suitable way to assess dynamically the performance and condition of a structure. Essentially, in the case of modal analysis for the estimation of the dynamic properties of a structure such as the grandstand terraces, the author was able to point out that experimental procedures alone might actually be inadequate due to the complex mode shapes a non-symmetric structure is likely to undergo. The author recommended the use of finite element analysis techniques as a primer to experimental procedures and he pointed out that the latter can produce highly reliable results.

The author could not stress enough the inclusion of boundary conditions in the analysis process as well as the importance of the correct choice of material properties and stiffness characteristics, hence recommending his procedures for any similar kind of analysis work

The author went on to alert the research community that the time is now opportune to encourage the Civil Engineering Industry to show confidence and trust and implement the finite element method as the most suitable companion and sometimes, even alternative to some complex and costly design and testing procedures and therefore benefit by saving themselves time and money.

4.6.4 Originality

The step-by-step manual FE-updating incorporating an „index of accuracy' linked to the updating procedure is both comprehensible and innovative. The analyst can have full control of his/her own method and at the same time have the benefit of full justification of the updated parameters. This is not always possible with the automatic updating methods whereby often, some updated parameters cannot be fully justified. The author was able to demonstrate that FEA can be highly versatile and accurate in extracting the modal parameters of a structure.

4.6.5 Value, Impact (citations)

This paper has been submitted in *Computers and Concrete,* An International Journal of high status, known for its innovations in computational methodology and application. It is however too early to test its value in terms of the number of citations attracted. It is hoped that the approach presented, as described above, will add to the confidence of practicing engineers in the use of finite element analysis and that it will have a good reception by the industry.

Finally, the author wishes to stress that he considers the remainder of his published work equally important, complementing the contribution he has made to the subject. Regrettably, due to limited space it is not possible to report it here. A small sample of selected citations is stated below such as the papers by Waldorf (2001) and Zhuge (2009) who reported repeatedly on the author's inspiring approach in teaching finite element methods.

Chapter 5

5.0 SYNTHESIS

To recoup, the main contribution of the author's research in the area of Computational Mechanics is the successful solution of complex problems in Civil and Structural Engineering by means of advanced and innovative computer simulations and numerical techniques. His research explores the extent to which costly and time consuming and sometimes fruitless laboratory, or full-size in-situ investigations, can interact with and ultimately be replaced by more cost effective, faster, beneficial and even more accurate, computer performed virtual tests.

The author has successfully modelled a number of different materials and structures with complex geometries, loading and boundary conditions, methodologies and behaviours. He has focused on reinforced concrete because it constitutes a relatively untouched and complex area in computational mechanics and revealed the essential niche. Hence, he was able to solve a series of engineering problems and make a modest contribution in the field of Applied Computational Mechanics. This is also substantiated in the author's individual publications in the following way:

5.1 **Paper I**, „A Numerical Model for the Computation of Concrete Pavement Moduli'.

The author began his research journey by representing soil and soil conditions in a rigorous mathematical way. He made a contribution to the academic world by developing a cubic spline function and used it alongside routine soil tests to model the behaviour of certain types of soils (Karadelis, 2000). In addition, he was able to generalise the above representation by introducing an exponential function for use when no data were available. He solved the problem of shear transfer between adjacent pavement structures by introducing the concept of mechanical efficiency in the algorithm, and he was able to reduce dramatically computational time and effort by embracing an axi-symmetric model as opposed to a solid, three dimensional representation. This approach had not been reported previously and is regarded as a key aspect of the author's contribution. The model was validated successfully using experimental and site data and it was used to extract information (layer stiffnesses) from within the pavement system and relate them to surface deflections.

With the aforementioned research the author has contributed to the disclosure and adoption of the Falling Weight Deflectometer (FWD) as a method for assessing and evaluating rigid (concrete) pavement structures. He has helped to make Local Authorities as well as engineering consultants and contractors in the UK aware of the potential and limitations of the FWD. Moreover, he has added to the knowledge in the area of Non-Destructive Testing and helped to improve the quality of roads for the benefit of road users. His research was sponsored by British Airports Authority (BAA) and Gatwick Airport and therefore these organisations benefited directly and explicitly from the research. The author's contribution is summarised as follows:

1. A successful mathematical representation of soil and other unbound layers within a pavement, using a cubic spline and an exponential function.
2. The solution of the problem of shear transfer in axi-symmetric representations by introducing in the algorithm the concept of mechanical efficiency.
3. The development of a non-destructive method for the structural evaluation of concrete pavements.
4. The disclosure of findings, advantages and shortcomings of the FWD to engineering consultants and contractors
5. The dissemination of knowledge to other researchers as it is entailed from his publications and citations.

5.2 **Paper II**, „Elasto-plastic FE Analysis with Large Deformation Effects of a T-plate Connection to Square Hollow Section'.

The author's experience in non-linear analysis has made him aware that sometimes it may be difficult to detect and recognise problems involving large deformation effects and allow for the latter in numerical representations. He demonstrated that the quality of the results obtained from such an analysis will not be affected if one has the large deformations option open, even if the latter do not, finally, take place. This option is highly recommended by the author when similar work (analysis of plates, struts, thin walled structures) takes place and especially when design optimisation procedures are sought. The author has pointed out that this technique should be particularly useful when revising existing, old design methods, or when design optimisation procedures take place, or when drafting or amending design guidelines, or even when a complete assessment of the strength of a particular type of structure or structural element is needed. The author's contribution due to this study is:

1. Improving the knowledge of the performance and accuracy of results of certain structures (plates, shells, beams) by introducing material and geometric non-linearities in the numerical analysis process.
2. Providing clear and rigorous guidelines for the optimum design of a family of structural connections.
3. Endow other researchers and practitioners facing similar problems, with useful ideas such as the large deformation effects, to be integrated in the analysis procedure.

5.3 **Paper III**, Grandstand Terraces, Part 1. Experimental Investigation.

This is an original example of experimental research as no other in-depth, full size laboratory investigation on grandstand terraces has been reported so far. The significance, value and benefits from full size, laboratory controlled tests compared to tests on scaled down, small specimens has always been well appreciated by researchers. Findings from the specific tests can be used directly to assist in drafting design guidelines and recommendations. They can be useful to other researchers and practitioners who have an interest and would like to make a comparison with similar structures of their own.

The author was able to enhance the understanding of the behaviour of these structures under different loading regimes. He used the experience gained to develop and „fine-tune' a series of successful computational models. He is confident that his recently published results will be utilised by a number of researchers who would like to build upon existing strengths. A summary of the author's contribution based on this research is as follows:

1. A novel experimental investigation of a family of grandstand terraces under loading-unloading regimes and the acquisition of good quality data for the designers of these precast units to revise their design philosophies.
2. Extracting stiffness values for the uncracked and cracked sections and using them to calibrate a finite element model as shown in the next article.

3. Underpinning established theories and use of the findings as a benchmark for further work by the author and other researchers on similar fields.

5.4 **Paper IV**, Grandstand Terraces, Part 2. Numerical Modelling

The author's general elasto-plastic, constitutive, meso-macro scale level approach, featuring cracking and crushing options of reinforced concrete has captured successfully the non-linear flexural behaviour of grandstand terraces, to failure. This model is useful to analysts who seek to look at reinforced concrete beyond the conventional codes of practice. It may epitomise an aid for the design of nuclear power installations, or offshore structures, or, in general, structures for specialised use such as silos or cooling towers. The author was able to diverge from the disputed area of strain softening and present a model based on more realistic failure criteria, that of cracking and crushing, treating concrete as a brittle material. In brief, he has made the following contributions:

1. A general elasto-plastic, constitutive, meso-macro scale level numerical model of a reinforced concrete structure, featuring cracking and crushing options for concrete failure and yielding for steel. This can be particularly useful to practicing engineers involved in the design of RC structures beyond the conventional codes of practice.
2. The same model was used to demonstrate and caution the engineering community that Crisfield's (1983) method developed specifically to replace Newton-Raphson's solution algorithm in special cases of non-linear analysis, does not always produce accurate results. In fact, it has been demonstrated that there is no such algorithm to-date that can be used to capture successfully the descending part of the stress-strain curve of any material.
3. Finally, the author suggested a simple but effective way to overcome the problem of contribution to shear resistance of the main reinforcement, by attributing it to the surrounding aggregate interlocking ability.

5.5 **Paper V**, „Grandstand Terraces. Experimental and Computational Modal Analysis'.

It was stated by the author that sometimes, carefully conducted computational modal analysis may produce more comprehensive results than the experimental one. He demonstrated that some complex modes of vibration, especially those associated with a relatively complex structure, may not be depicted by the available testing hardware and software. He suggested that complex experimentation should be accompanied by a rigorous finite element analysis in an effort to identify fully the resulting dynamic behaviour of a structure.

Opinions have been divided regarding the influence of reinforcement in the dynamic properties of concrete structures. Interim studies by the author hinted towards the possibility that the introduction of, or an increase in, the amount of reinforcement is likely to force the structure into a different mode of vibration, hence altering the previously obtained properties. The author has suggested that the above needs more dedicated work so that possible boundaries can be established and the general behaviour can be postulated. Changes in dynamic properties may not always be detected when increasing the amount of reinforcement in a structure alone, because any change in the dynamic response of the system takes place gradually and because there is nothing to be used as a benchmark and therefore distinguish and compare the different modes obtained by the change in reinforcement. Nevertheless, the author has made the engineering community aware of the possible effects of the reinforcement in the dynamic properties of a structure.

The author is aware that testing or modelling parts of a structure alone, does not necessarily provide a global understanding of its dynamic performance. He has noticed that at certain modes of vibration, the system of two terrace units under observation responded independently and the motion of one unit had the tendency to damp the motion of the other. Currently, his efforts are headed towards developing a suitable pattern to predict the performance of the entire structure by studying certain key parts of the latter. The main arguments of his contribution are:

1. It is highly recommended that experimental modal analysis is always accompanied by FEA for a better understanding of the dynamic properties of a structure.
2. The influence of reinforcement in the dynamic performance of RC structures has been sending out conflicting signals. The author has indicated that the reinforcement can change the dynamic performance of a structure by reverting from one mode of vibration to another.

3. The author has cautioned the engineering community that by studying parts of a large structure such as a grandstand, one cannot draw reliable conclusions regarding the dynamic performance of the whole structure itself.

5.6 **Paper VI**, Reliability Pointers for Modal Parameter Identification of Grandstand Terraces

A critical discussion about the extraction of eigenvalues and eigenvectors was within reach by the author after experimenting with different solution algorithms. He concluded that the Block Lanczos algorithm performed significantly better than other solvers by reducing the CPU time and providing more accurate results. Hence the author did not hesitate to recommend this solver for similar types of analysis work.

He carried out an effectual updating procedure and was able to test the validity and usefulness of the restrains offered by the code used. He noticed that the overall performance of the units was highly reliant on the stiffness of the supporting structure and in particular on the elastomeric bearings (pads) inserted between the concrete unit and the supporting steel frame. Hence, he developed a stiffness matrix simulating the shear and compressive stiffnesses of the pads and introduced this information in the general solution algorithm. He was able to witness a dramatic improvement in the accuracy of the results and recommend his findings to other researchers for similar work.

Also, he noted the importance of the material properties in the correlation procedure and commented on the significant improvement achieved when the latter were revised accordingly. In addition, he introduced a verification process aiming to strengthen the accuracy of his numerical and experimental findings but recognised that even an “exact” solution may not sometimes improve confidence because one may not have full control of all the parameters in the equations developed.

Finally, he concluded that in some cases, results from a rigorous finite element model can be more accurate and easily interpreted than corresponding results obtained from experimental techniques. Hence, he called upon the civil engineering world to make more use of the finite element method. Essentially, his contribution from this research is summarised as follows:

1. A recommendation to use the Block Lanczos eigenvalue extraction method as it appears to perform a good deal faster than any other method and produce more accurate results.
2. A proposal accompanied by strong evidence to include modelling of supports (boundaries) in any rigorous FE structural analysis and especially in the dynamics domain.
3. A recommendation to adopt the finite element model updating procedure described in the article, as the method for extracting the dynamic properties of structures.

Finally, the author's main contribution to his field of interest is the numerical representation of engineering structures and the solution of engineering problems encountered, by means of numerical or computational methods. His chief aim is to introduce the appropriate knowledge and confidence, reduce the amount of costly experimental (laboratory and site) work and replace it, with virtual tests.

Ultimately, the author is fully aware of the limitations of his work, dealt with in the next chapter. Virtual testing necessitates enormous confidence disseminated and shared by his peers. Also, he is endeavoured to continue his research by constantly improving his numerical models and by looking into new areas as highlighted in Chapter 7.

Chapter 6

6.0 CONCLUSIONS

The author has been using computational mechanics and essentially the Finite Element Method to solve engineering problems for a very long time. During the last ten years he has focused his research interests towards the particular area of concrete, and concrete structures. He has identified mathematical and physical trends, attitudes and opinions and has analysed them critically with respect to their engineering validity, usefulness and practicality. He has developed his own critical hypotheses, on the performance of reinforced concrete alone, or in combination with other structural materials and under both static and dynamic actions and has attempted to question and contest arguments generated by others in favour of implied limitations or ‚automatic' solutions. These ideas he has disseminated through peer reviewed scientific and engineering journals. The latter have been accepted by fellow academics and engineering practitioners and have helped to shape the development of this area of engineering and computational mechanics.

The papers representing the author's portfolio of evidence in this specialised field of Computational Mechanics have been supplemented by a number of additional articles. A number of them have been published in internationally prominent journals, or proceedings, or appeared in national conferences, workshops, lectures and presentations. Some have made a contribution to technical/scientific reviews and discussions in well-known journals, such as the *Proceedings of the Institution of Civil Engineers*. All the above can be found in the author's CV submitted with this document. The author has also tried to pass the knowledge gathered from his research to his students and other fellow academics by blending it with his lectures and publishing in pedagogical journals (Karadelis, 1998, 1999). As a consequence, this output has earned him a credible reputation as a research scholar specialising in the above mentioned field and has reserved a place for him in both, the academic and engineering practice communities.

Each of these studies, as analysed in the author's main portfolio of evidence, makes a considerable contribution in improving the understanding of reinforced concrete behaviour as a structural material. They also contribute significantly to the area of computer simulation of structures. The first paper provided an example of research that generated a robust numerical model that was firmly grounded in finite element analysis. This work was of significant practical value to engineering professionals, the public and the non-destructive testing specialists. The next study was an important springboard in applied structural modelling work and a good example of successful numerical representation of a structural steel connection. This work improved the numerical image and behavioural accuracy of certain types of structures (plates, shells, beams).

The next two papers examined the under-researched area of reinforced concrete as a material, highlighted the advantages and drawbacks of all the state-of-the-art numerical representations and concluded by proposing a rigorous but practical numerical model for wider use. It was maintained that such a model may have a practical value, not only to the construction industry but also the research community. The novel experimental investigation of grandstand terraces can be regarded as the stepping stone for further computational analysis of reinforced concrete structures. This investigation should have considerable practical value not only because of the resulting computational model but also for revising the design approach of these units.
The aforementioned experimental investigation led the author in developing a novel global numerical model for reinforced concrete. This innovative paper demonstrated that it is reasonable (and more realistic) to ignore the highly disputed descending part of the typical stress-strain performance curve of concrete (strain softening) and base the numerical model on its more realistic brittle behaviour. The last two papers exhibit a different characteristic to all previous studies as they concentrate on studying the basic dynamic (as opposed to static) behaviour of the same concrete terrace units mentioned earlier. These types of studies can be used as an aid for the design and construction of new structures that can be susceptible to vibrations. They explore rarely recognised problems of structural behaviour using specific methods and theories. They convey a message to fellow academics, engineering analysts and specialists that experimental modal analysis alone may not adequately portray the dynamic properties of a structure.

Some of the papers constituting the author's auxiliary portfolio surfaced from commissions by the private industry to investigate particular aspects of their work in order to improve productivity (Karadelis & Brown, 2000), or by professional organisations (Chapman & Karadelis, 1994) to study and report on construction markets abroad. A few papers disseminate the author's build-up research experience to teaching. They have been cited and viewed as very constructive and informative by peers and students (Karadelis, 1998). Finally, several papers were spin-offs from a wider dissemination of his research through international conferences. Collectively, a strong case can be argued that all these papers have made a significant contribution to research in the UK engineering community in the area of Computational Mechanics.

The author hopes that the bulk of work presented above demonstrates his wider analytical and computational skills. The author has established and demonstrated an in-depth ability to research from within a designated area and through various studies. It is likely that he has made and presented a successful critical analysis based on his own material and other sources. He has applied successfully a range of research methodologies and has used the finite element method of analysis extensively to test both experimental and physical models. As a result, he has been able to develop some original approaches to data modelling during this time. The conclusions of the various studies have provided insights and recommendations that have been useful to academics, engineering practitioners and students.

Chapter 7

7.0 FUTURE

The author has been developing, using and assessing numerical analysis models and looking into their validity, applicability and usefulness through a fulfilling research period of over 10 years. For instance, he developed a successful non-destructive evaluation technique capable of predicting the structural condition of concrete pavements. In a different case, he developed a successful numerical model capable of predicting the modes of failure of reinforced concrete structures and later their dynamic properties (natural frequencies & mode shapes). In addition, the author has identified and reported on numerical solution techniques which are not really applicable to such a wide spectrum of structural problems as it was initially claimed (Crisfield's arc-length method) and has been able to comment on the influence of reinforcement on the dynamic properties of reinforced concrete structures and revise existing inaccuracies.

The author is well aware of the limitations of his own research and in particular the margins within which his numerical models operate. He knows that he has not, as yet, addressed fully the implications of these limitations and their impact on the numerical simulation world. For instance, the effective computer representation of the failure mechanism of the bond (debonding), between steel reinforcement and the surrounding concrete at meso/macro scale level, is still to be addressed and he is certain that this should improve the quality of his results. Also, the computer representation of the complex loads signifying human activities remains unanswered. If the latter is resolved the performance of grandstands under popular activities like jumping, stamping and beating would be easier to study. Consequently, the author's future research and publications plans include further and more focused research into the following areas:

1. The continuous development of Green Overlays, a specially developed concrete pavement repair management system (CPRMS), an overlay material and method for the structural repair of damaged concrete pavements. The author has won an EPSRC grant and has recruited a research student who is currently working on a specific part of the topic. In addition, two, internally funded students have joined

the project team, working on similar topics. The author is currently preparing a new research proposal this time to recruit a research assistant, who will be overseeing the design and development of the PRMS for the British road and airfield networks.

2. The ongoing research on Grandstand Vibrations and the challenge to continue with a series of probabilistic types of analyses, simulating crowd generating loads is to be addressed. In order to deal with the high demands of this project and predict all vibration characteristics of these structures, a series of numerical models are linked together in an effort to deliver the expected results. So far, indications signal towards a series of complex loading regimes, such as Monte Carlo simulations (Amar, 2006), a technique used to approximate the probability of certain outcomes by running multiple trial runs (simulations) using random variables, which seem to model the behaviour of football/rugby supporters adequately.

3. In addition, the author will continue to consider new, novel ideas and areas of applied research. One such area is the investigation of noise and vibration generated from building mounted micro-wind turbines. Initially, this would involve the development of a hypothesis for the quantification of vibration and its effect on the structure and its occupants. The hypothesis will be tested through laboratory/field investigations on a statistically valid sample size. Careful consideration will be given on the likely range of mounting methods, types of building constructions and configurations available in the UK, to provide a sufficient range of results. The research should provide the appropriate understanding for the reduction and gradual elimination of noise and vibrations.

The articles represented the author’s portfolio of evidence in the specialized field of computational mechanics have been supplemented by a number of other articles published in internationally acclaimed journals or refereed conference proceedings. As a consequence of these publications the author is regarded as one of a relatively small number of recognized scholars in this specialised area of mechanics with a bias in reinforced concrete. The author has been asked to sit in the review panel of a number of journals such as Structures and Buildings and Engineering Sustainability of the ICE, Construction and Building Materials and others.

In conclusion, the author has accomplished a sustained and systematic improvement over time, resulting in his own development and contribution to knowledge and practice. His portfolio of evidence suggests that he has made a rightful contribution in the area of computational mechanics and specifically a considerable and original contribution in the area of modelling reinforced concrete. The author's aspiration would be to be considered for a higher degree of PhD by Portfolio on that basis.

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Appendix I

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A numerical model for the computation of concrete pavement moduli: a non-destructive testing and assessment method

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Abstract

The falling weight deflectometer (FWD) non-destructive testing technique has been used to monitor and assess the behaviour and performance of rigid pavement systems. In addition to the full-scale site investigation, tests were also carried out with the aid of a specifically developed laboratory scaled model of the FWD.

A rigorous finite element model was developed to analyse a multi-layered pavement system with various material and geometric properties and to relate the surface deflections as measured to the computed values. Evidence of non-linearity and deviation from classical linear elastic theory led to a more complex mathematical solution to fit the experimental data more accurately. The laboratory and field test results were compared with the computed values. This paper includes extensive discussion of these results and the conclusions drawn from them.

Keywords: Falling weight deflectometer; Young's modulus; Pavement quality concrete

1. Introduction

The falling weight deflectometer [1–3] was first introduced in France about 30 years ago to test the flexible road networks. It has since gained increasing acceptance as one of the most effective methods for evaluating flexible roads. Recently, it has also been used to quantify the condition of joints in concrete pavements and detect deterioration in cement treated layers below the surface [4]. Essentially, a weight impacts the pavement and the deflections are measured by a series of seven geophones: one at the centre of the impact plate and six at other positions equally spaced along a radius. A heavy version of the FWD can produce a maximum instantaneous dynamic force, as measured by a load cell of up to 250 kN, to simulate one wheel of a fully loaded Boeing 747. The impact time is between 20 and 25 ms.

1.1. Objectives

The primary research objectives were as follows: first, to measure simultaneously under a given load the surface deflections and the stresses and strains in the layers of a pavement; and second, to correlate these results with theoretical values obtained from a finite element analysis, for the same loading pattern, using experimentally determined elastic modulus values for each layer. If the correlation between theoretical and experimental stress, strain and deflection values could be achieved, then the assumed moduli should be representative of particular pavement materials.

An infinitely rigid slab would theoretically distribute concentrated loads to the full extent of its boundaries and would not deform. However, in its simplest form a rigid pavement refers to a concrete slab resting on one or more soil layers and it is called 'rigid' because the modulus of elasticity of the slab is several hundred times greater than that of the underlying soil.

The overall objectives of the investigation were as follows:

- Examine the suitability of the FWD-system for assessing rigid pavements in general and a specific multi-layered rigid pavement airport site in particular [1,3,5].
- Examine the possibilities of regarding pavement layer stiffness as an index of the structural condition of the pavement, by reviewing the existing methodology [6–9].
- Examine the importance of relevant parameters such as critical stresses and strains within the layers of the pavement, the moisture content, the duration of loading, the geometry of the pavement structure, the drainage conditions and the stress history [10].
- Relate the stiffness mentioned above, to the surface

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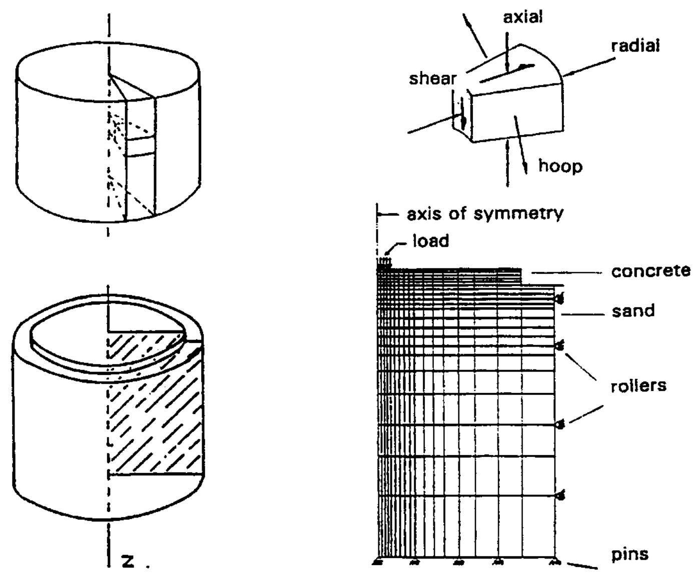

Fig. 1. Cylindrical solid subjected to an axi-symmetric load and corresponding FE-model, featuring two-dimensional isoparametric quadrilateral elements.

deflection under a known load, or other readily measured parameter of the pavement.

- Prepare a long-term airport pavement site instrumentation plan, capable of providing useful structural information of the pavement conditions and at the same time carry out similar laboratory tests.

Originally, the research programme consisted of four major parts as follows:

- Development of a theoretical model to assist with the usual parametric and sensitivity studies.
- Auxiliary experimental work and possible development of a small-scale laboratory model.
- Long-term full scale monitoring/tests at Gatwick Airport.
- Validation of the method, by comparing theoretical and experimental results and by testing known pavements sites.

2. The numerical model

Existing analytical methods of modelling rigid pavement structures were examined [11–14] and after studying the predicted behaviours under loading, detailed consideration was given to the development of a rigorous finite element model capable of analysing a multi-layered pavement system with different material properties. The mathematical idealisation of the pavement system was attributed to a "cylindrical solid subjected to an axi-symmetric load" as defined by Timoshenko and Goodier [15] (Fig. 1).

2.1. A linear elastic approach

The assumptions associated with the above model included homogeneous and isotropic conditions for the materials as well as a constant Young's modulus for each layer. Each element had to satisfy both compatibility and equilibrium conditions with adjacent elements or, depending on the geometry and symmetry of the model, with a number of boundary nodal points. The deformation of the cylindrical solid under the action of a load acting through its axis of symmetry is also symmetrical about the same axis. Hence, only a "wedge" cut out of this solid needed to be analysed (Fig. 1). Further, by considering the thickness of this wedge as linearly varying from zero to unity, the three-dimensional problem was reduced to a two-dimensional plane strain one thus simplifying further the analysis. Tangential displacements do not occur and stresses and strains do not vary in the tangential direction. The problem of load transfer due to dowel bars at the joints of the concrete pavement was addressed by applying the concept of mechanical efficiency such as:

$$n = \left[\frac{\delta(\mathrm{u})}{\delta(\mathrm{l})} \right] 100$$

where $\delta(\mathrm{u})$ is the displacement of the unloaded slab at a joint, $\delta(\mathrm{l})$ the displacement of the loaded slab at the same joint, and n is the efficiency in percentage.

A typical efficiency (load transfer) of 80% was assumed and was introduced to the model as a prescribed displacement. The rectangular mesh was selected to be denser for the top part of the structure and around the axis of symmetry where high stress concentration was expected. Mesh boundaries were introduced at a sufficient distance from the axis of loading for edge effects to be insignificant. The structure was assumed sufficiently deep for stresses and strains to reach negligible values during the analysis. Negligible Young's modulus values were given to those parts of the soil structure that tended to develop tension characteristics. Finally, a two-dimensional isoparametric, quadrilateral, plain stress, axi-symmetric finite element was used in the analysis.

2.2. A non-linear approximation

However, after evidence of non-linearity and deviation from the linear elastic theory, the soil properties were reviewed. The difficulty of simulating soil behaviour is that soil does not obey classical plasticity assumptions on which limit state theory is usually based In addition, it shows at all stages of loading an irreversible strain, thus violating elasticity laws. As a result, a cubic spline function was introduced to fit an experimental set of stress/strain data and later, an exponential function was developed to represent soil behaviour where no such data were available.

The cubic spline function shows below, it short matrix notation, has the advantage that it provides continuous first and second derivatives and therefore is very suitable for finite element procedures.

$$S(\varepsilon) = \{N\}\cdot\{q\}$$

where $S(\varepsilon)$ is a spline function w.r.t strain, $\{N\}$ the vector of the interpolation function and $\{q\}$ the vector of generalised nodal co-ordinates.

In order to fit a set of cubic functions (cubic spline) through a set of points, the following conditions must be met:

(a) each interval between consecutive data points must be represented by a different cubic function;
(b) the slope of any pair of cubics that join at a data point must be the same;
(c) the curvature of these cubics at the particular point must be the same.

The resulting system of equations containing second and first derivatives as unknowns was solved using iterative technique and the spline was fully defined. Hence, as the behaviour of the soil was represented by a stress curve depicted from triaxial tests, only the experimental data on this curve were input into the program. The initial tangent moduli were found by differentiating the spline with respect to strain, such as:

$$E_t = S'(\varepsilon) = \{N'\}\cdot\{q\}$$

where $\{N\}$ and $\{q\}$ are the column vector functions of σ and ε

The magnitude of the initial tangent stiffness was found later to have a very significant influence on the load deformation behaviour under incremental loading.

Based on triaxial test results the following exponential function was developed:

$$y = a\{1 - e^{-sx/a}\}$$

where s and a, are the non-zero constants.

The above function possesses some useful properties for numerical representations.

(a) It always passes through the origin.

$$(x, y) \equiv (0, 0)$$

(b) The slope at the origin can always be specified.

$$\frac{dy}{dx} = s\,e^{-sx/a} = s$$

(c) As x increases the curve approaches the horizontal line $y = a$, asymptotically.

$$\lim_{x\to\infty} y = a \qquad \text{and} \qquad \lim_{x\to\infty} \frac{dy}{dx} = 0$$

(d) The slope always decreases steadily.

$$\begin{aligned} &\text{from}: \quad dy/dx = s \quad \text{at the origin} \\ &\text{to}: \quad dy/dx = 0 \quad \text{at } y = a \end{aligned}$$

The above function has the advantage that it is very simple to define in terms of slope and upper limit limiting value. In addition, a direct comparison with the triaxial test results gave a very good correlation [16]. Such a function represented this particular type of soil well.

2.3. The incremental procedure and the initial tangent modulus

The total load was divided into partial load increments, which were added one at a time. During the application of each increment were assumed to be linear. The initial stiffness matrix was used to generate the equations for the next increment and so on, until the process was completed. This is, an appropriate modulus value was assigned to each element at the onset of the application of each new increment.

Incremental displacements were added together to give the total displacement at any stage of loading. Stresses and strains were treated in the same manner. The procedure was repeated until the total load was reached. The material parameters were then computed from the stress–strain curve obtained. Plasticity conditions were also defined in the program for the soil medium. That is, when loading the soil was undergoing plastic as well as elastic deformations (strains) dictated by the spline function. The type of plastic material was defined by the von Mises yield criterion (strain energy stored in an element of material is energy due to both change in volume and shape).

The soil started to yield when the first load increment was applied. The load to cause yield was estimated from a previous elastic run with a unit load applied on the structure and with stresses restricted to the critical stresses of the material; the latter being estimated from the laboratory tests.

The advantage with the incremental procedure is that initial stress or strain may easily be introduced. The method

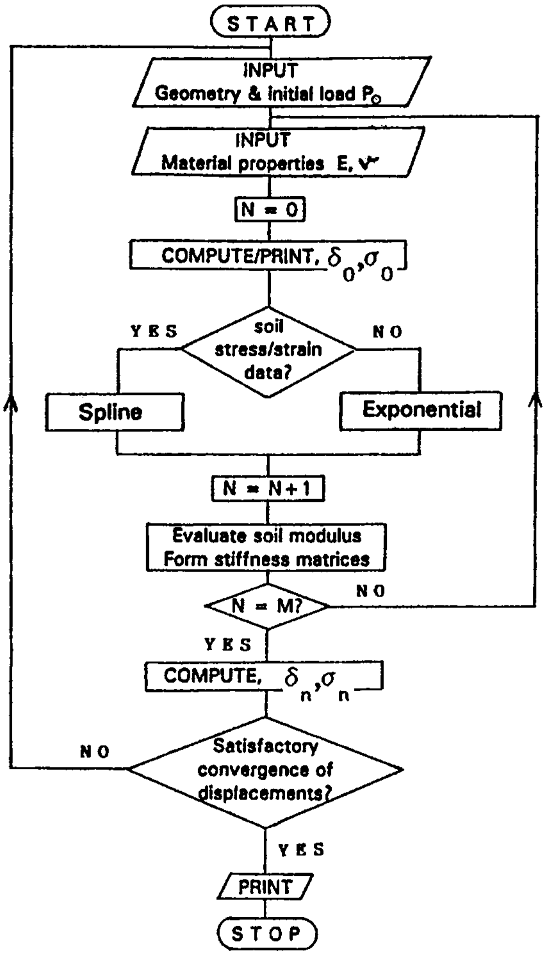

Fig. 2. Simplified pavement evaluation procedure.

can also model some plastic behaviour. However, what it cannot model is the strain softening; in order to simulate a stress decrease beyond a peak, it would require a negative modulus value, which the finite element method cannot cater for.

A general form of the non-equilibrium equation is shown below:

$$\{F^e\} = [k^e(\delta, F)]\{\delta^e\}$$

where $[k^e(\delta, F)]$ is the element stiffness matrix function of $\{\delta\}$ and $\{F\}$, $\{\delta^e\}$ the displacement vector of the element, and $\{F^e\}$ is the load vector of the element.

A flow chart for the pavement moduli evaluation procedure was developed, which is shown in the abbreviated form in Fig. 2. Clearly, for any given deflection bowl, more than one set of moduli could yield 'best-fit' results. The procedure as shown in the flow-chart depends upon the initial modulus value selected for each layer. Hence, re-adjustment of the modulus, values, possibly using back analysis techniques [17] are unavoidable if better agreement with the experimental surface displacement is required.

3. Results and discussion

The following were observed in the laboratory and were also validated by the FE-model. Careful study of the acceleration/time history of the loaded slabs indicated that the response to the falling mass, as expected, with the appearance of the first cracks. Loss of contact with the soil around the circumference of the slabs (curling of the edges) was also noted. This was caused by vertical, out-of-phase, oscillation of all points of the slab along a radius. The ratio D/T (diameter to thickness) dictated the deformed shapes of the slab and the mode of failure due to impact. For small D/T ratios the predominant mode of failure was punching shear, whereas for large D/T ratios the failure mode was essentially in bending.

Reasonable agreement was obtained between the results from the finite element analysis and those recorded in the laboratory. The correlation between the theoretical and experimental soil pressures was generally good and is shown in Fig. 3. Also, very satisfactory was the agreement between the theoretical and experimental strains at the base of the concrete slab, as shown in the same figure. The central accelerometer, housed within the load cell, did not withstand the impact and ceased to function after two or three drops. It was not possible, therefore, to obtain central deflection values. It is clear from Fig. 3 that if the central displacements had been obtained, the measured displacements downwards would have been considerably less than the computed displacements for the assumed E-values at all positions. This would appear to imply that higher E-values would have been more appropriate and this is discussed again later.

No long-term monitoring or measurements at Gatwick airport could be made as intended but the deflections of specific pavements for which some information was available were measured with a FWD-machine. Fig. 4 shows the results along with the theoretical displacements obtained from the FE-analyses for the set of properties shown in Table 1.

Using the theory for the conical load distribution beneath a loaded plate [18,19], a simple sensitivity analysis was performed. According to this theory for a three-layer pavement system, the third layer influences mainly the extreme surface deflections, D6 and D7 of the FWD. The intermediate layer influences the corresponding D3, D4 and D5 deflections and the top layer controls the central (maximum) deflections. This appears appropriate for flexible pavements for which it was developed but it may, not be entirely satisfactory for rigid pavements where the concrete slab acts as a much better load-spreading medium.

Layers 2 and 3 were considered identical and were adjusted together. Layer 1 for the pavement quality concrete (PQC) was kept constant at the static value of 30 GN/m^2. The procedure in the computation was to adjust in sequence layers 2 or 3, and then layers 4 and 5. The run times for the computer which then available were unfortunately

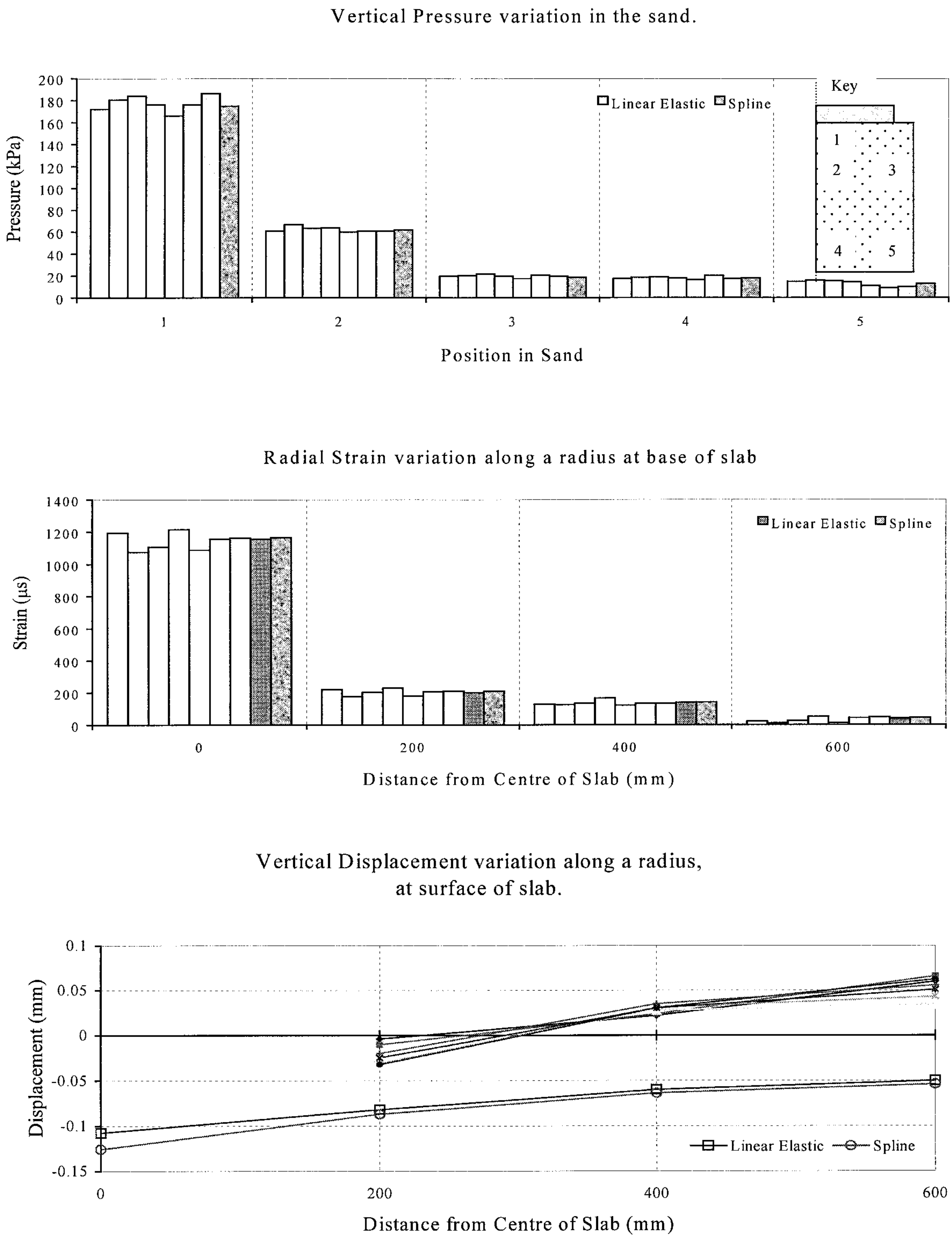

Fig. 3. Comparison between theoretical and laboratory results for impact load of 35 kN.

inordinately long and severely limited the number of FE analyses which could be made. The values shown for E_1 were obtained at an intermediate stage and those for E_2 were the values when further adjustment was discontinued.

Similarly, the sensitivity analysis indicated that although the values for layers 2 or 3–5 changed fairly dramatically from E_1 to E_2, the theoretical surface displacements changed relatively little at all locations. This is shown in Fig. 4. It is clear therefore that the static E-value of 30 GN/m^2 taken throughout for the PQC (layer 1) was too low and that a higher, dynamic E-value should have been assumed and then adjusted as necessary for the best fit. In addition, while the conical load distribution theory provides a useful guide in the case of rigid pavements, the influence of the

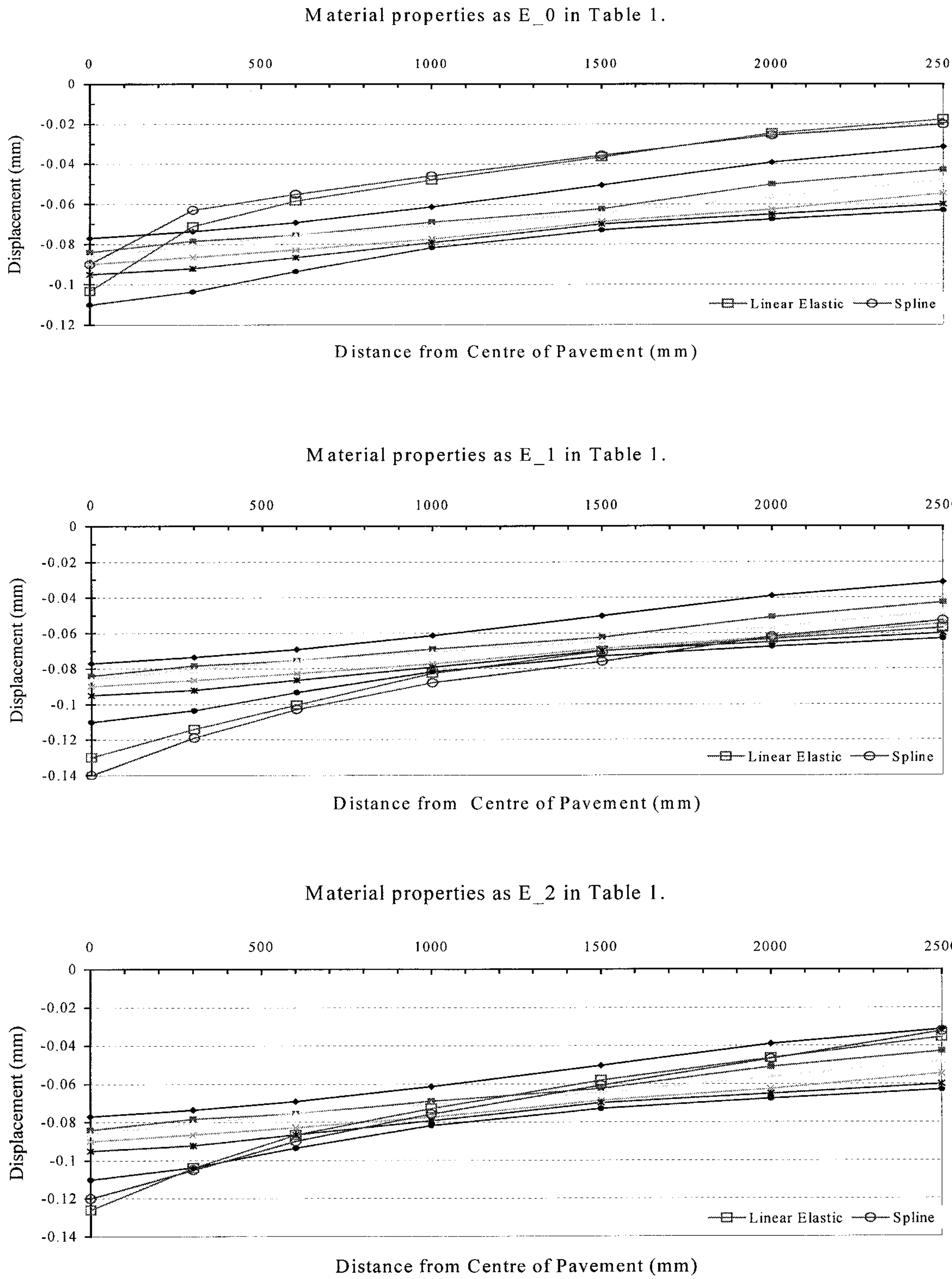

Fig. 4. Comparison between theoretical and in-situ surface displacements for FWD load of 100 kN at Gatwick Airport.

PQC is much greater and more widespread than layer 1 in a flexible pavement.

Comparing the surface displacements shown in Fig. 4 due to very large changes in layers 2 or 3 from E_0 to E_2 confirm that the significance of this layer can also be large. With regard to the lower layers then although dynamic values should also apply for soils, it is clear that even a large increase in these values will not provide a large change in the outer surface displacements.

Table 1
Material properties (E_o, ν) as assumed initially and as adjusted theoretically for rigid pavement construction at Gatwick Airport

Layer no.	Description	Modulus (MPa)			Thickness H (mm)	ν
		E_0	E_1	E_2		
1	Pavement quality concrete	30 000	30 000	30 000	400	0.25
2	Lean concrete	20 000	150	2000	150	0.25
3	Lean concrete	20 000	150	2000	150	0.25
4	Compacted crushed stone	30	3	25	300	0.35
5	Gatwick soil	20	10	20	8000	0.40

4. Conclusions

The following conclusions can be drawn:

- Reasonable correlation between observed and theoretical deflection bowls can be obtained by assigning appropriate E-values to the corresponding pavement, sub-base and soil layers. The correlation can then be improved considerably by adjustment of the E-values for successive layers for the theoretical FE-model. The top PQC layer should be adjusted first to improve the fit at the centre and then improve successively at locations further out, in accordance with the conical load distribution theory, by progressing sequentially downwards through the layers.
- Further cycles from top to bottom can be repeated as above to achieve an optimum solution based on the best-fit concept.
- The instantaneous, dynamic E-values should be used for all the layers, especially the Pavement Quality Concrete. This would have given much better agreement between the surface displacements obtained from the FE-analyses and the observed displacements for the present results.
- Non-linear soil representation improved the accuracy of the predicted surface deflections significantly. However, this increased the computation time and it is probable that even for the lean concrete and soil layers, non-linearity is relatively small for the dynamic relationships. Again, this emphasis the need for as accurate a representation of the material properties as possible for the initial selection.

It is apparent that the existing methods for rigid pavement evaluation and assessment require further development. However, the present experimental and theoretical work indicate that it is possible to relate the layer stiffnesses to the surface deflections of a pavement system and hence to treat the latter as an indirect index of the deterioration of the pavement. Further work is required to improve the application of the method for a quality assured FWD-technique within a Pavement Management System.

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Elasto-plastic analysis with large deformation effects of a T-end plate connection to square hollow section

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Abstract

The behaviour and performance of a family of structural connections made of square hollow sections (SHS) has been investigated in the laboratory and a series of data have been collected and presented in a graphical form. In parallel, a rigorous finite element model was developed capable of analysing the system of the SHS, the cap-plate, the cleat-plate and its surrounding weld. Evidence of non-linearity and deviation from the classical linear elastic theory led to a more complex numerical solution to fit more closely the experimental data. Hence, a specific methodology is presented, as it applies to analyses involving plasticity and large deflections (deformations). Test results obtained in the laboratory were compared with computed values from the finite element analysis and are presented graphically in the last pages of this paper. Satisfactory agreement was obtained between recorded and computed strains and displacements. The paper includes extensive discussion of the above results and the conclusions drawn from them. A brief account of directly related future research work is also given. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: SHS; ANSYS; Material non-linearities; Large displacements

1. Introduction

There is no doubt that the description of non-linear phenomena inevitably lead to non-linear equations which immediately render classical methods of mathematical analysis inapplicable. No method is yet known for finding the exact solution to a system of non-linear equations, such as the one shown below:

$$\{F^e\} = [k^e(\delta, F)]\{\delta^e\}, \tag{1}$$

where $[k^e(\delta, F)]$ is the stiffness matrix of the element which is a function of $\{\delta\}$ and $\{F\}$, $\{\delta^e\}$ is the displacement vector of the element and $\{F^e\}$ is the load vector of the element.

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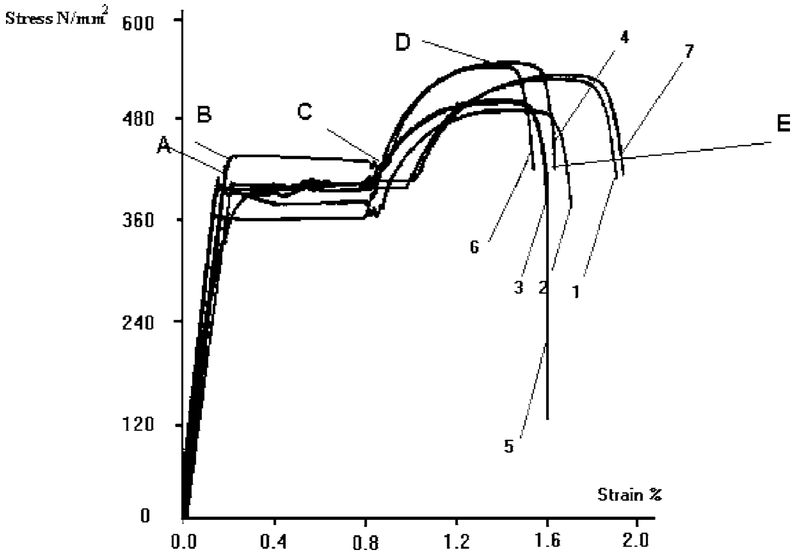

Fig. 1. Typical stress strain curves obtained from tensile tests on seven specimens, cut out of rolled SHS.

Non linear structural behaviour arises from a number of phenomena, which can be grouped into three main categories such as, changing status, geometric non-linearities and material non-linearities.

First Reimer et al. [1] studied the behaviour of complex, welded tubular joints using finite element analysis techniques and compared their results with a similar photo-elastic model. They suggested that the computer program used, must have efficient non-linear solution algorithms for the model to be considered successful and concluded that by far the least accurate modelling is the use of thin shell elements.

Kitipornchai et al. [2] developed the element stiffness matrix for an angle and a T beam-column section incorporating geometric non-linearities and compared their results with independent numerical solutions. They were able to demonstrate the need of the geometric updates and proposed further, more accurate studies.

A family of structural connections made of $80 \times 80 \times 4$ square hollow sections (SHS) has been the subject of investigation in the Civil Engineering laboratories of the University of Coventry. All specimens were provided by British Steel and the material used was designated as grade 50 high yield steel ($\sigma_y \approx 340\,\mathrm{N/mm^2}$) suitable for building and other structural uses. Typical stress strain curves were obtained by running tensile tests on seven specimens, cut out of an equal number of rolled SHS (Fig. 1). All tests were carried out at normal environmental conditions (room temperature) [7].

2. Geometric non-linearities

If a structure undergoes large displacements as the load is applied incrementally, then the stiffness matrix will not be constant during the loading process. For instance, when a small tensile

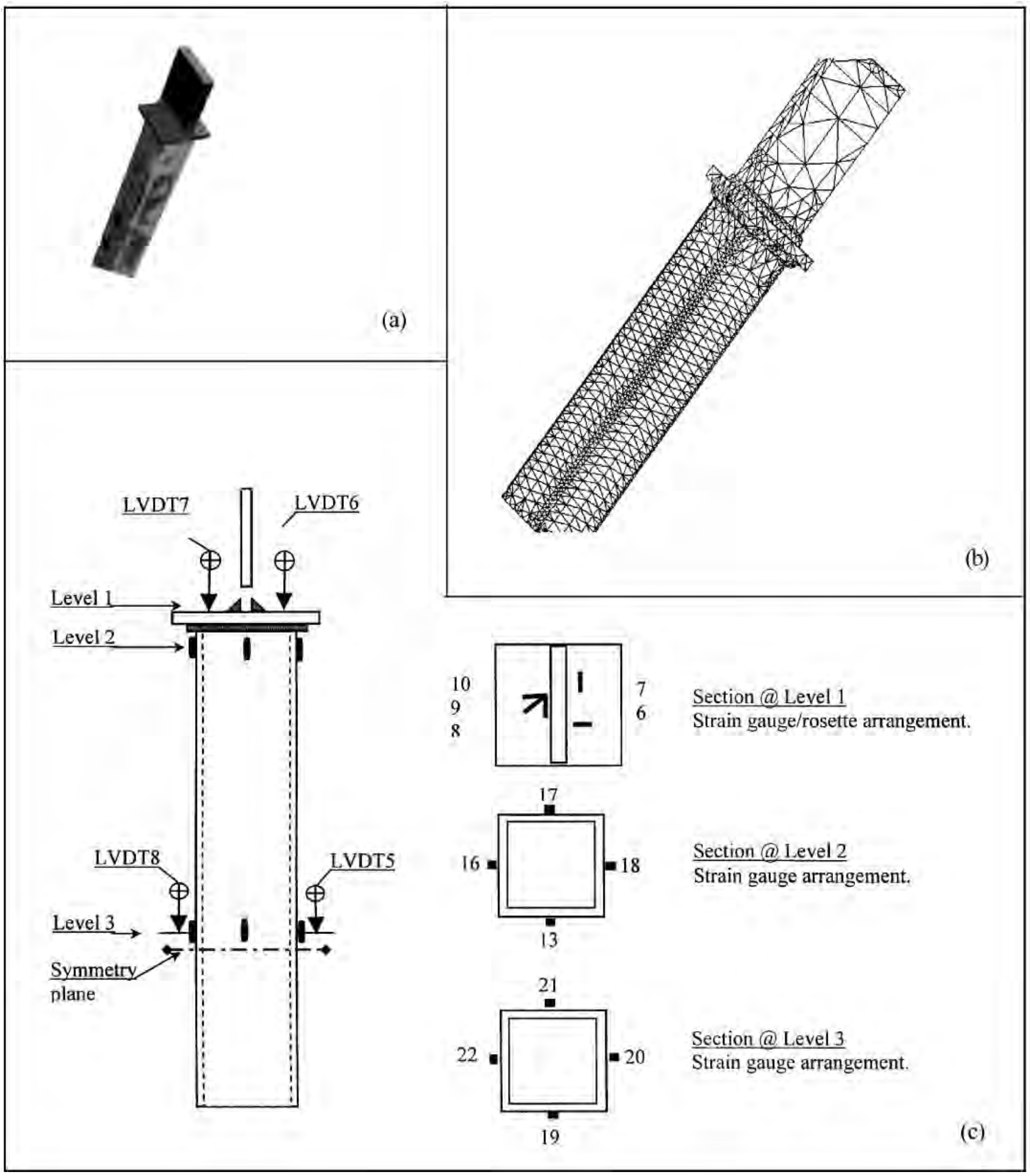

Fig. 2. (a) View of the real specimen; (b) finite element model featuring 3D tetrahedral elements and (c) line diagram of the model showing position of transducers.

load is applied at the centre of a cap-plate welded around the periphery of a square hollow steel member (Fig. 2), the strain energy stored in the material is due to buckling of the plate only. However, when the load becomes large enough to bow the plate significantly, the area of the plate around the line of application of the load will undergo further deformation to accommodate the additional strain. Hence, the stiffness of the plate at this central region will be expected to increase.

ANSYS [3] activates the large deflection analysis within the static analysis domain using the *NLGEOM,ON* option. This can be summarised as a three step process for each element, such as:

1. Determination of the updated transformation matrix $[T_n]$ for the element.
2. Extraction of the deformation displacement $\{u_n^d\}$, from the total element displacement $\{u_n\}$, in order to compute the stresses and the restoring force $\{F_e^{n-r}\}$.
3. After the rotational increments in $\{\Delta u\}$ are computed, node rotations are updated.

Any desired loading can be applied. In a typical case like the above, the total load was divided into seven sufficiently small increments (dictated by laboratory tests) and these were applied one at a time. In order to obtain valid solutions at intermediate load levels, and/or to observe the structure under different loading configurations, seven load steps were introduced in the analysis. The Newton–Raphson (N–R) [4] equilibrium iterations drive the solution to equilibrium convergence (within some tolerance) at the end of each load increment. That is, before each step solution, the N–R method evaluates the out-of-balance (o-o-b) load vector, $(\{F_{\text{restoring}} - F_{\text{applied}}\})$ and checks for convergence. If the set criteria are not satisfied the o-o-b load vector is re-evaluated, the stiffness matrix is up-grated and a new solution is attempted. The iterative procedure continues until the problem converges. If convergence cannot be achieved then the program proceeds to the next step and tries again.

ANSYS stresses that relying solely on displacement convergence can result in gross errors and recommends force convergence checking. Hence, the default (force) convergence criteria were used. The program performs a check for force convergence by comparing the Square Root of the Sum of the Squares (SRSS) of the force imbalances against the product of $VALUE \times TOLERANCE$ [3], where $VALUE =$ SRSS of the applied loads and $TOLERANCE = 0.001$.

Hence, the analysis was organised as follows:

1. A total of seven load steps were defined over a "time" span.
2. The program was instructed to perform several solutions within each load step (sub-steps) and therefore the total load was applied gradually.
3. The first load increment of 245 kN was followed by an additional six of 10 kN each to reach a total load to cause failure, of 305 kN.
4. A number (maximum 25 per sub-step) of equilibrium iterations were performed for a converged solution to be achieved.

ANSYS strongly recommends that for large displacements analyses the loads specified in the load steps should be 'stepped up' (as opposed to 'ramped on'). This means that the value of a particular load step will be reached during the first iteration and will be kept constant during the remaining iterations, until the end of the load step. This contributes to faster convergence. After each increment the deflections caused were calculated by using the linear version of the Eq. (1), above. That is, it was assumed that the stiffness matrix was constant during the application of each load increment. The initial Tangent Modulus was taken from a stress–strain curve obtained from experimental observations. The initial stiffness matrix $[K_0]$, was then computed from the tangent modulus and the Poisson's Ratio. The initial stiffness matrix was used to generate the equations for the next increment and so on, until the process was completed for that particular load step. It is important to keep the load increments small, so that the increments in displacement cause

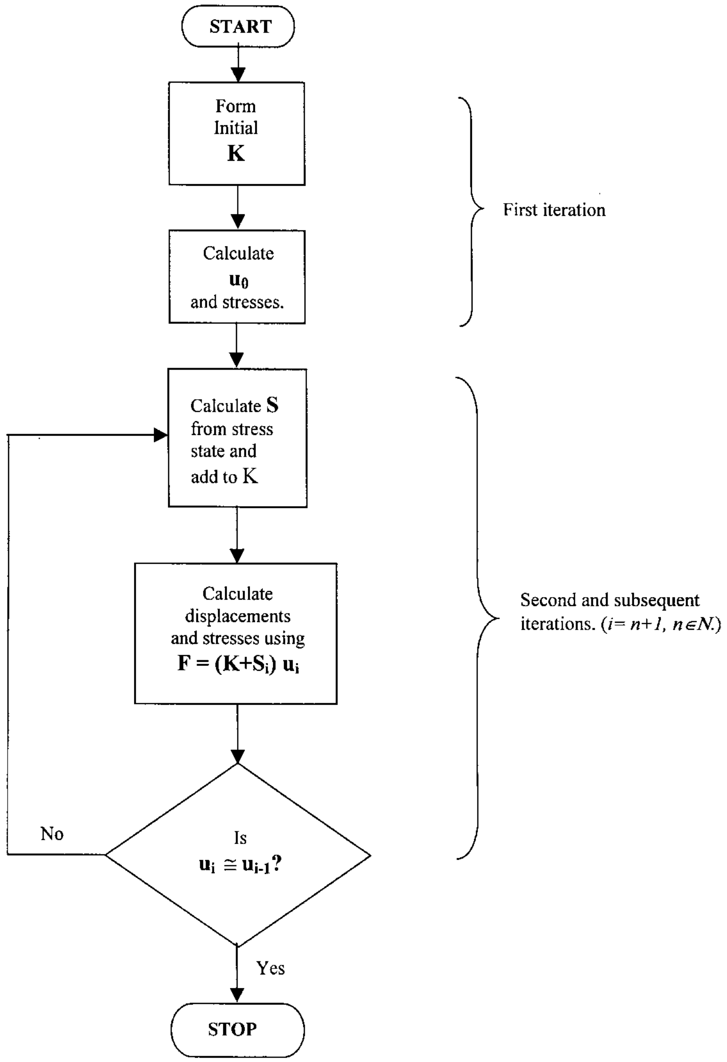

Fig. 3. The basic steps of a non-linear, large displacements analysis.

negligible changes in the stiffness matrix at each load step. The original co-ordinates of the nodes were then shifted by an amount equal to the values of the displacements calculated. The new stiffness matrix for the deformed plate was re-calculated and the process was repeated until the total load was reached. The matrix notation for the incremental procedure, using the linear version of Eq. (1), is:

$$\{\Delta F_i\} = [K_{(i-1)}]\{\Delta\delta_i\}, \quad \forall\, i \in N^+ : i = 1, 2, 3, 4, \ldots \tag{2}$$

where i is the positive integer representing stage of incremental loading.

A synoptical flow chart containing the main steps for large displacements analysis and demonstrating stress stiffening effects after the second iteration, is shown below, in Fig. 3. Simply stated, stress stiffening looks at the state of stress in a FE-model and calculates a stiffness matrix, $[S]$, based upon it. Matrix $[S]$ is then added to the usual (initial) matrix $[K]$ and a new set of displacements are calculated.

3. Elasto-plastic behaviour

Material non-linearities occur when the stress is a non-linear function of the strain. The relationship is also path dependant, that is, the stress depends on the strain history as well as the strain itself. The general theory for elasto-plastic analysis, provides the user with three main elements: The yield criterion, the flow rule and the hardening rule [5].

The yield criterion determines the stress level at which yielding is initiated. The flow rule determines the direction of plastic straining (i.e. which direction the plastic strains flow) relative to x, y, z axes. Finally, the hardening rule describes the changes the yield surface undergoes with progressive yielding, so that the various states of stress for subsequent yielding can be established. For an assumed perfectly plastic material the yield surface does not change during plastic deformation and therefore the initial yield condition remains the same. However, for a material experiencing strain hardening, plastic deformation is generally accompanied by changes in the yield surface. Two hardening rules are available and these are isotropic (work) hardening and kinematic hardening.

3.1. Multilinear isotropic hardening

For materials with isotropic plastic behaviour, the assumption of isotropic hardening under loading conditions postulates that, as plastic strains develop, the yield surface simply increases in size while maintaining its original shape. For metals, ANSYS, recommends the von Mises yield criterion with the associated flow rule and isotropic (work) hardening. When the equivalent stress is equal to the current yield stress the material is assumed to undergo yielding. The yield criterion is known as the 'work hardening hypothesis' and assumes that the current yield surface depends only upon the amount of plastic work done.

The solutions of non-linear elastic and elasto-plastic materials are usually obtained by using the linear solution, modified with an incremental and iterative approach. The material is assumed to behave elastically before reaching yield as defined by Hooke's law. If the material is loaded beyond yielding, then additional plastic strains will occur. They will accumulate during the iteration process and after the removal of the load will leave a residual deformation.

In general:

$$\varepsilon_n = \varepsilon_{n(\mathrm{ela})} + \Delta\varepsilon_{n(\mathrm{pla})} + \varepsilon_{n-1(\mathrm{pla})}, \tag{3}$$

where ε_n is the total strain for the current iteration, $\varepsilon_{n(\mathrm{ela})}$ is the elastic strain for the current iteration, $\Delta\varepsilon_{n(\mathrm{pla})}$ is the additional plastic strain obtained from the same iteration, $\varepsilon_{n-1(\mathrm{pla})}$ is the total and previously obtained plastic strain.

Convergence is achieved when $\Delta\varepsilon_{n(\mathrm{pla})}/\varepsilon_{n(\mathrm{ela})}$ is less than a criterion value (the default for ANSYS is 0.01) [6]. This means that very little additional plastic strain is accumulating and therefore the theoretical curve, which is represented by a series of straight lines (multi-linear approach) is very close to the actual one.

For the case of uniaxial tension, it is necessary to define the yield stress and the stress–strain gradients after yielding. When σ_{zz} becomes greater than the uniaxial yield stress, then yielding takes place. The yield condition is the von Mises yield criterion for one dimensional state of stress:

$$\sigma_{zz} \geqslant \sigma_{\text{yield}}. \tag{4}$$

A set of flow equations (flow rule) can be derived from the yield criterion. The associative flow rule for the von Mises yield criterion is a set of equations called the incremental Prandtl–Reuss flow equations, [5]. That is, the strain increment is split into elastic and plastic portions. Hence, it is necessary to apply the total load on the structure in increments. These load steps need only start after the FE-model is loaded beyond the point of yielding. The size of the subsequent load steps depends on the problem. The load increments will continue until the total load has been reached or until plastic collapse of the structure has occurred. As the load increases the plastic region spreads in the structure and the non-linear problem is approached using the full N–R procedure. The stiffness used in the N–R iterations is the tangent stiffness and reflects the softening of the material due to plasticity. It should be noted that in general, the flatter the plastic region of the stress–strain curve, the more the iterations needed for convergence.

As plasticity is path dependant or a non-conservative phenomenon, it requires that in addition to multiple iteration per load step, the loads be applied slowly, in increments, in order to characterise and model the actual load history. Therefore, the load history needs to be discretised into a number of load steps with the presence of convergence tests in each step. ANSYS recommends a practical rule for load increment sizes such as the corresponding additional plastic strain does not exceed the order of magnitude of the elastic strain. In order to achieve that the following procedure was followed:

Load step one was chosen so that to produce maximum stresses approximately equal to the yield stress of the material. The yield stress was estimated from the experimental stress–strain curves. The load to cause yield was validated by performing a linear run with a unit load and by restricting the stresses to the critical stress of the material. This was found to be approximately equal to 240 kN. Successive load steps were chosen such as to produce additional plastic strain of the same magnitude as the elastic strain or less. This was achieved by applying additional load increments no larger than the load in step one, scaled by the ratio E_T/E. Such as:

$$P_{n+1} = \frac{E_T}{E} P_n \quad \forall n \in N, \tag{5}$$

where is the E = Elastic slope and E_T = Plastic slope, with E_T/E not less than 0.05

Table 1 summarises the plasticity theory that characterises the elasto-plastic response of a certain type of materials. However, the basic steps characterising a non-linear elasto-plastic analysis are shown in a brief flow chart in Fig. 4.

4. The finite element model

The basic model consisted of the rectangular hollow section with the cap-plate fully integrated at one end and the cleat-plate on top (Fig. 2). As the structural member (tie) was symmetrical about

Table 1
Summary of the theories involved in a material with multi-linear isotropic hardening behaviour

Material option	Yield criterion	Flow rule	Hardening rule	Material response
Multi-linear Isotropic Hardening	von Mises	Associative (Prandtl–Reuss equations)	Isotropic	Multilinear

a plane at right angles to its longitudinal axis, only half of the member was initially modelled (further symmetry was considered at a later stage). Material properties such as a Young's Modulus value of 205000 N/mm^2 and a Poisson's ratio of 0.27 were inserted in the program. Translational restraints were applied at the cut end and a negative pressure load (tension) of was applied at the cleat plate.

The correct choice of element is very important in finite element analysis. A 3D, four node tetrahedral structural solid element (SOLID 72) with three translational and three rotational degrees of freedom (DOF) per node was chosen. This is described by ANSYS as a general purpose element particularly suited to automatic meshing of irregular volumes. A linear elastic stress analysis was performed, and the results were normalised for other load values.

Utilising the experience obtained from the laboratory tests and the linear elastic analysis results, the model was divided into three substructures. Previous experimental work with various cap-plate thickness values demonstrated that when thin, the latter undergoes excessive deflections under and near the bearing of the cleat plate. Hence, the cap plate was allowed to undergo large displacements and was given a non-linear stress–strain representation (material non-linearities). The region of the SHS near the cap-plate was seen to develop excessive stress concentration. These stresses tended to exceed the yield stress of the material, hence non-linear material properties were attributed to it. Finally, at this early stage of the investigation the weld was given the same properties as the SHS.

A non-linear (multi-linear, elasto-plastic) analysis featuring isotropic hardening effects was performed. The large displacements option was kept open to accommodate possible non-linear effects of the cap-plate. The results are shown plotted in the following figures.

5. Results and discussion

Fig. 5 shows the variation of strain with load measured and calculated just below the interface of the cap-plate and the hollow section. This is a region of high stress concentration as predicted by the finite element model. Strain output between strain gauges 13 and 17 as well as 16 and 18 was very similar, as expected (see Fig. 2, diagram of test specimen). In order to save space, these strains were averaged and plotted in pairs. A plot from the non-linear analysis is shown superimposed for ease of comparison. The same procedure is repeated for strains developing at positions 19 and 21 and also 20 and 22, measured 300 mm below the cap plate on the SHS body and presented in Fig. 6. The finite element analyses results are also plotted with them. It can be seen that the strain values

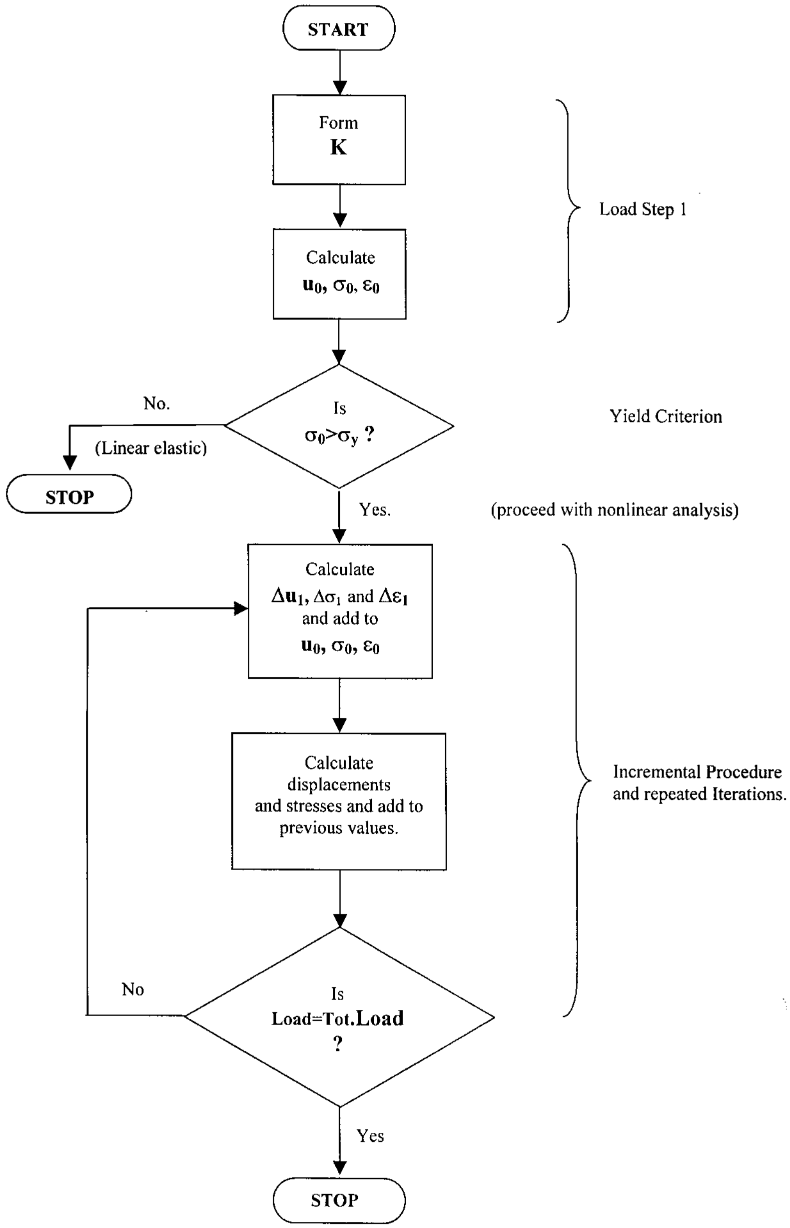

Fig. 4. The basic steps of a non-linear, elasto-plastic analysis.

predicted by the non-linear finite element model are in good agreement with the corresponding results obtained in the laboratory.

Fig. 7 shows the variation of displacements as measured at positions 5 and 8 and also 6 and 7. The agreement here cannot be considered as satisfactory as the one above. A percentage of the error involved is attributed to the ageing DENISON machine on which some experimental tests

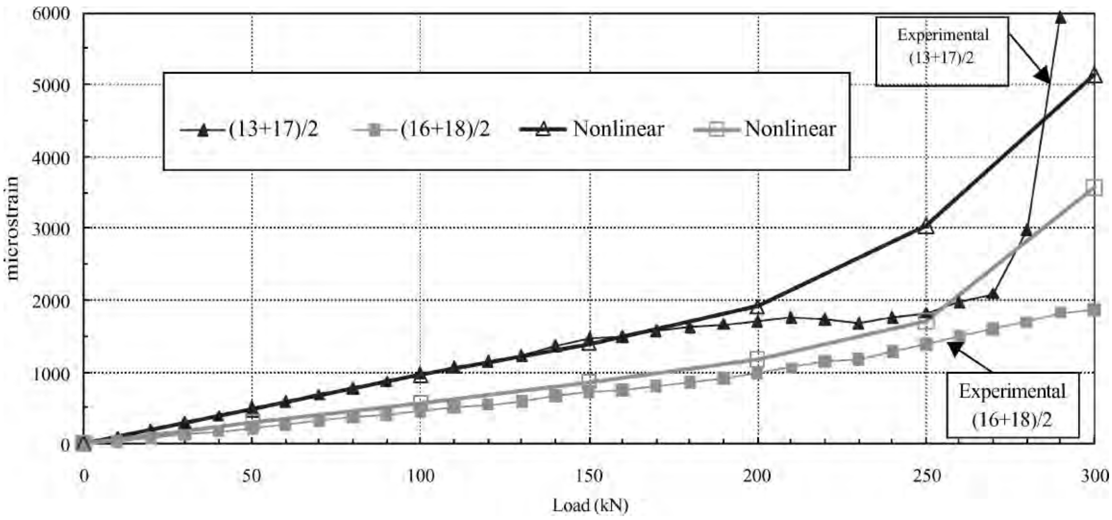

Fig. 5. Variation of experimental and calculated average strain with load strain gauges: (13 + 17), (16 + 18).

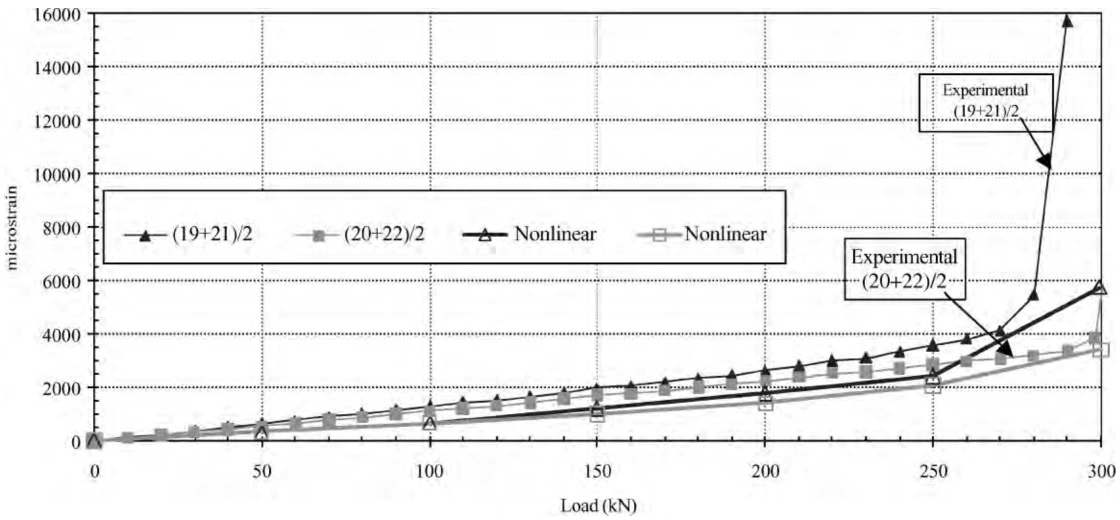

Fig. 6. Variation of experimental and calculated average strain with load strain gauges: (19 + 21), (20 + 22).

were carried out. Indeed, a series of simple tensile and compression checks were performed and produced different results. It was found that although there was good correlation with the load cells used as checks in all compression tests, correlation was not acceptable for the tensile tests. It was later revealed that the machine is calibrated annually by an external body, in compression only. Hence, accuracy was not present when tensile tests were performed. It was estimated that the error involved was of the order of 11.6% in the linear region. Unfortunately, it was not possible to

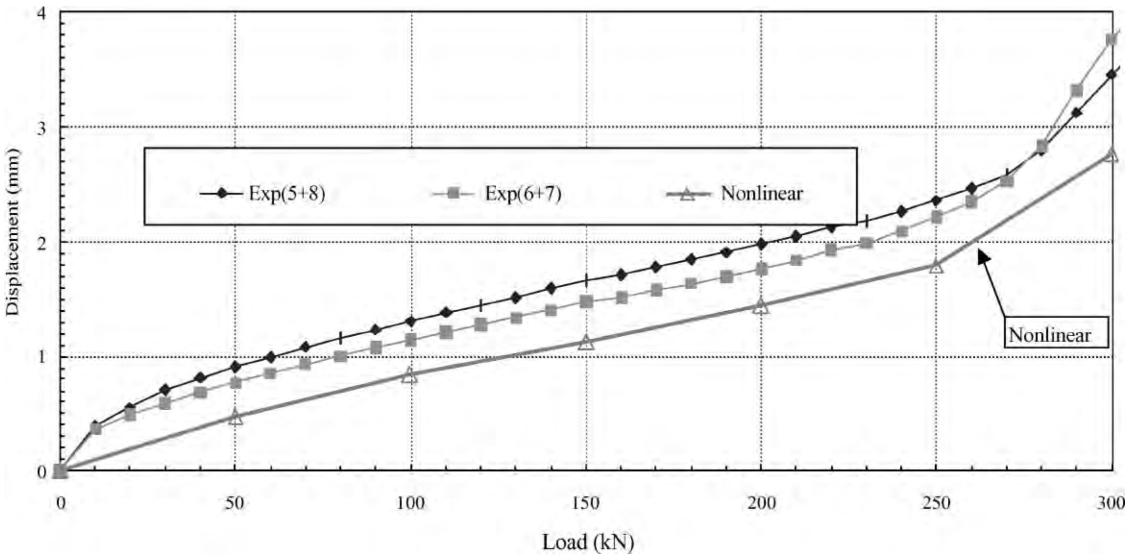

Fig. 7. Variation of average displacement with load. LVDTs: (5 + 8), (6 + 7).

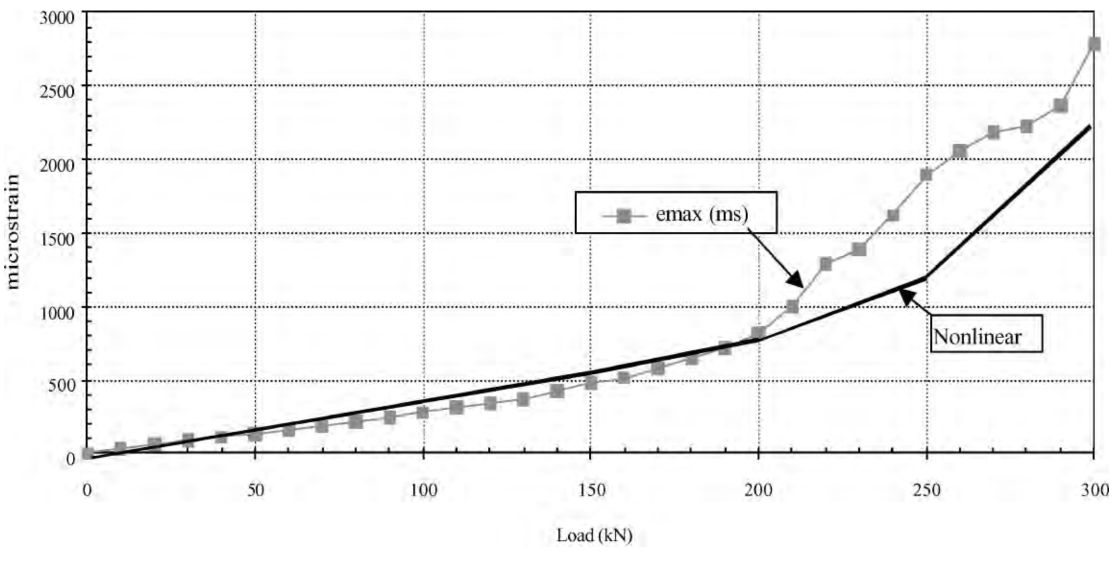

Fig. 8. Variation of maximum principal strain with load position: strain rosette 1,2,3 @ cap plate.

estimate the corresponding error in the non linear range and therefore no attempt has been made to adjust the results.

Fig. 8 shows the variation of maximum principal strain with load, as measured at level 1, on the cap plate. Strain and hence stress distribution, in this region is complex as the plate is undergoing out of plane bending and tension. Laboratory results are plotted alongside those from the finite element analysis. It can be seen that the predicted results are in good agreement with the experimental ones.

6. Conclusions and future work

A finite element model has been developed capable of depicting the behaviour of a T-end plate connected to SHS. Satisfactory agreement was obtained between experimental and calculated strains at three different levers on the SHS connection. The deformed shape of the cap-plate, noted in the laboratory, was also predicted accurately by the FE-model. Stress concentrations were also predicted to develop directly under the cap-plate, below the edges of the cleat plate, indicating symptoms of yielding of the weld, possible partial separation from the parent metal and therefore the necessity to model the weld more accurately.

The discrepancy between the displacements recorded in the laboratory and those computed by the FE model was noted. However, a careful study revealed an error with the calibration of the tensile load apparatus.

The finite element model is currently undergoing refinement, updating and more rigorous calibration. Efforts will be concentrated in representing more accurately the weld around the cap-plate and between the interface of the cap-plate and the cleat-plate. The revised model will be used to carry out a series of parametric and sensitivity studies to assist with the development of the appropriate design guides.

Having performed a large number of both, linear and non-linear analysis runs, it is appropriate to state that the ANSYS FEA code can easily model materials such as steel, especially in the linear region. It is apparent that an accurate representation in the non-linear region depends a great deal on the code user and the choice he/she can make over a large number of parameters. Deep knowledge of the engineering principles combined with a good understanding of the code is essential for the safe and efficient use of any finite element program. Based on the present results it can be said that the current non-linear model can depict the non-linearities of the material and can be useful in predicting the performance of the structural member under tension.

However, one drawback of the current finite element representation is its inability to model possible separation between the weld and the parent metal. It is anticipated that had the welded regions between the interface of the cleat plate and cap plate and also between the cap plate and the SHS body, been modelled in a more rigorous manner, their contribution to the better behaviour of the SHS connection would have been enhanced. The later is currently under investigation. Modelling of the welded regions has been approached from a different angle. As localised thermal stresses develop at very short time due to welding, temperature dependent properties are introduced and a thermal analysis is coupled to the structural analysis to compensate for any residual stress effects. In addition, non-linear contact elements have been selected to allow for possible separation of the SHS components. Finally, curve fitting techniques are employed to model material behaviour even more accurately. The results, so far, look very promising.

The platform used was a PENTIUM II, desk top computer with 120 Mbytes of RAM memory. The non-linear analysis was usually set to run during the night as it took several hours to produce results.

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Concrete grandstands. Part I: experimental investigation

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Loading–unloading tests were carried out on uncracked (as delivered from the factory) and cracked (after the first loading–unloading cycle was completed) grandstand terrace units. The variation of parameters, such as displacements and strains, with the applied load was recorded and presented in a graphical form. The reduction in stiffness of the units owing to cracks was estimated from these graphs. The predominant mode of failure was found to be cracking initiated at the soffit of the units (tension zone) and mainly around the symmetry line (where maximum bending stresses congregate). These cracks propagated gradually towards the top. The measured and predicted strain distribution across the depth of the vertical part of the terrace unit (riser) was found to be predominantly linear, displaying tension at the bottom and compression at the top. A large portion of the horizontal part of the unit (tread) followed closely the behaviour of the riser, however, to reveal tension rather than compression at the top. This could have some implications for the design of the units. It was concluded that present methods and procedures of evaluating and designing precast concrete terrace units are not integral. Further tests are required, coupled with more analytical work. A Part II companion paper reports on the development of a numeric algorithm describing the analysis process.

NOTATION

F load
$\boldsymbol{k}$ stiffness
δ displacement

1. INTRODUCTION

The most common construction of sports stadia today is that of a hybrid type where precast concrete terrace units span between inclined (raker) steel beams and rest on each other, thus forming a grandstand (Figure 1). The role of the third (resting) support is to stop the units from undergoing excessive twisting and, in general, provide extra stability. Accurate analysis and optimum design of these prefabricated units (elements), as well as the grandstand as a whole, requires a good understanding of their behaviour and performance under static and dynamic loading. Optimising their structural sections and improving economy, safety and comfort in use, is an ongoing engineering challenge with industry and academia working in tandem.

In particular, the applied loads and load mechanisms generated by sports and music fans on grandstand and other stadium structures are not yet fully understood or benchmarked. Codes of practice in the UK and abroad are not as rigorous and informative as they should be. Understanding the influence of these loads on grandstands would necessitate short- and long-term investigations on suitable structures. Their mathematical 'reproduction' would require complex numerical techniques, based on and supported by experimental findings.[1]

The current paper is part of an ongoing research programme at Coventry University, aiming to extend the understanding of the structural behaviour of sports stadia assembled from interconnected precast concrete units, by providing all those interested with a rigorous interpretation (numerical modelling) of the behaviour of these structures, initially under static loads (published as a seperate part II paper)[2] and later under dynamic actions.

2. EXPERIMENTAL PROGRAMME, METHODOLOGY AND PROCEDURES

A series of comprehensive laboratory tests were carefully planned and executed. The aim of this preliminary investigation was twofold. First, to examine the behaviour of a family of reinforced concrete (RC) structures supported at three positions and undergoing static, incremental loading. Second, to estimate the uncracked and fully cracked stiffness of the units. Two tests per unit were carried out for the latter. Test 1 assumed the section uncracked, as it was delivered from the factory; test 2 considered the same section, this time fully cracked, as received from test 1. Three 'identical' units made of

Figure 1. A precast concrete grandstand

the same batch were tested to failure. The vast majority of the data collected, correlated well. For reasons of clarity, only the results from unit 1 will be reported here.

The L-section terrace units were designed, manufactured and transported to Coventry University. Owing to limited space in the laboratory the smallest actual size was ordered. They were approximately 4·8 m long, encompassing a 700 mm wide by 100 mm thick horizontal member (tread) and 150 × 275 mm upstand (riser), as per Figure 2. Their properties are shown in Table 1.

Effective depth, d = 230 mm
K-factor = 0·091
Lever arm factor = 0·886
Lever arm, z = 205·8·8 mm
Depth to neutral axis, s = 76·6 mm
Area of tension steel required, A_{st} = 258 mm^2
Tension steel provided = 1T20 (314 mm^2)
Area of compression steel required, A_{sc} = 0 mm^2
Compression steel provided = 1T12 (113 mm^2)

Table 3. Output details

The design was based on BS 8110[3] and produced the results shown in Tables 2 and 3. It is clear from these tables that, for design purposes, the units were considered simply supported at the ends only and analysed as spanning the long dimension. Constructional details show the units spanning between two steel raker beams with two steel 'stools' welded on the rakers providing the necessary platform under the riser. Each unit is propped along the front edge by the riser of the lower unit.

The raker beams were not reproduced in the laboratory for obvious reasons. Instead, two steel 'stools' were placed under the riser, spanning 4·5 m apart. A suitable UB-section was placed under the front edge imitating the riser of the lower unit on site. Elastomeric bearings (neoprene pads) were inserted between the two materials as per actual conditions.

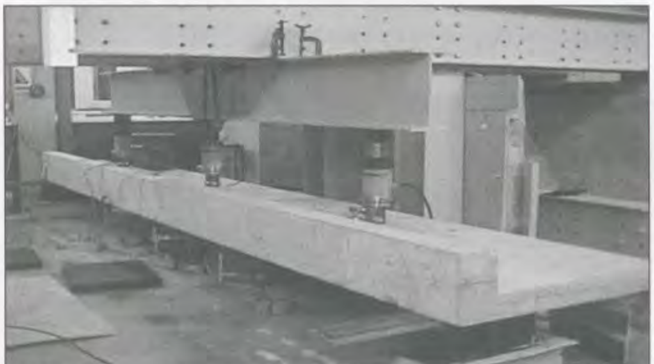

Figure 2. A precast concrete terrace unit as set up for testing in the laboratory

The heavy structures area of the civil engineering laboratories, comprising a strong floor and an array of rearrangeable steel stanchions and beams, was utilised. Six concentrated loads, which were equal in magnitude, simulated a uniformly distributed load (UDL) and were applied on the tread at 700 mm centres (Figure 2), using hydraulic jacks and spreader beams. The line of action of the UDL was parallel to the riser and at a clear distance of 100 mm from it. The load was applied incrementally and was kept constant during the collection of data. Loading and unloading tests were performed for 'uncracked' and 'fully cracked' units. The following parameters (variables) were measured using a series of appropriate transducers shown schematically in Figure 3.

(*a*) The maximum displacement was measured at the centre of the unit, under the riser and at two other symmetrical positions, to ensure symmetrical behaviour.
(*b*) The surface strain of the longitudinal tension (bottom) reinforcement of the riser, using electrical resistance strain gauges (ERSGs).
(*c*) Also, ERSGs were used to measure the strain at the lateral bottom reinforcement of the riser/tread.
(*d*) The concrete surface strain distribution across the depth of the riser section, using four pairs of demec points.
(*e*) The concrete surface strain at seven other positions on the unit, using demec points and a set of mechanical (analogue) strain gauge dials.

It was envisaged that the above measurements should provide a good understanding of the behaviour of the terrace unit.

Material properties	Loading	Cover to reinforcement
Characteristic concrete strength, f_{cu} = 45 N/mm^2	Load type: Uniformly distributed load	30 mm at sofit and the sides of riser and 40 mm at the top of tread
Reinforcement (T&C) characteristic strength, f_y = 460 N/mm^2	Dead load (self-weight) = 3·650 kN/m^2	
Reinforcement (shear) characteristic strength f_{yv} = 460 N/mm^2	Imposed load = 4·000 kN/m^2	

Table 1. Material properties, loading and cover to reinforcement

Serviceability state	Ultimate limit state
Reactions: $R_1 = R_2$ = 12·056 kN Maximum bending moment = 13·563 kN m @ mid-span	Reactions $R_1 = R_2$ = 18·139 kN Maximum bending moment = 20·406 kN m.@ mid-span

Table 2. Forces and moments

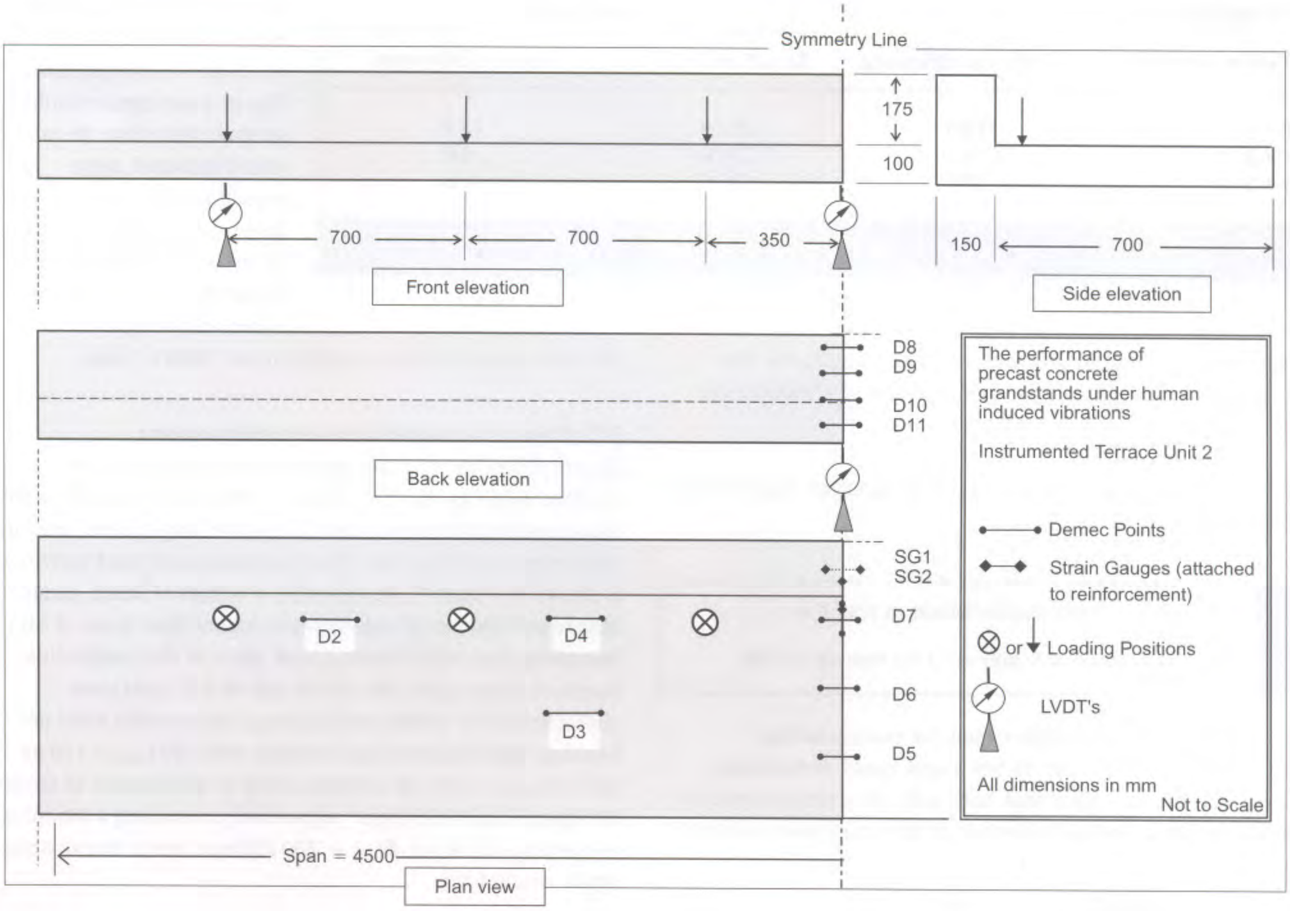

Figure 3. Terrace units 1 and 2, main reinforcement and instrumentation layout

3. RESULTS AND DISCUSSION

3.1. Displacements

Figure 4 shows loading and unloading paths of maximum displacement measured at mid-span using linear variable differential transducer, (LVDT 2) for uncracked and fully cracked units. The same figure, showing the performance of the units in terms of their deflection, can be used as an index of their conformity. Tests 1 and 2, 3 and 4, and 5 and 6 refer to units 1, 2 and 3 respectively. Odd numbers denote tests on uncracked units.

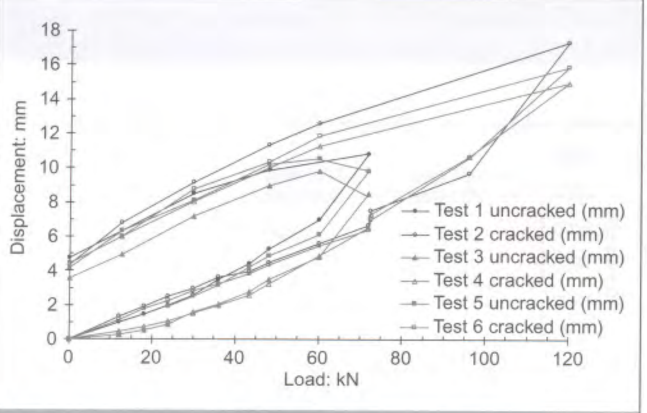

Figure 4. Terrace units 1, 2, 3. Tests 1 and 2 (uncracked and fully cracked units). Comparison between maximum displacements (LVDT2). Loading–unloading

Units 1 and 3 are in good agreement while unit 2 shows a little variation. Table 4 summarises and presents the above in terms of their percentage difference. All differences were based on deflection values related to maximum loads reached—that is, 72 kN for the uncracked and 120 kN for the cracked units.

It is evident that the path described by curve test 2 is smoother and not characterised by any sudden 'strain jumps', until after it exceeds the maximum value of 72 kN met in test 1. Beyond this load further cracking takes place, producing another 'strain jump' and permanent deformation.

Up to a load of 30, possibly 32 kN, all slopes are similar, approximating linear behaviour and showing good correlation. This compares well with the ultimate design load of 36 kN allowed by the designer. After the first initial cracking in test 1 (above 30 kN), however, the two curves diverge. The displacement of the uncracked section becomes noticeably higher compared to that of the cracked. At 60 kN the corresponding displacements for the uncracked and cracked sections were 6·98 and 5·6 mm respectively. At 72 kN (the maximum load allowed for the uncracked unit) the corresponding displacements were 10·75 mm and 6·5 mm.

Considering that the fully cracked unit has suffered some permanent deformation owing to loading at test 1, it is not surprising that its displacement values are now lower than those of the uncracked section. The maximum displacement

Uncracked units		Cracked units	
Test X and test Y	Percentage difference	Test X and test Y	and age difference
1 + 3	18·0	2 + 4	13·2
1 + 5	9·2	2 + 6	8·0
3 + 5	10·7	4 + 6	5·6

Table 4. Percentage difference in terms of deflection between units 1, 2 and 3

reached by the cracked unit was 17·25 mm at 120 kN. The permanent displacement after load removal was found to be approximately 4·5 mm in both tests.

Adding residual displacements to test 2, inherited from test 1, would yield

$$\text{Total displacement at test 2} = 4{\cdot}5\ \text{mm} + 6{\cdot}5\ \text{mm} = 11{\cdot}00\ \text{mm @ 72 kN}$$

This is only marginally higher than the corresponding displacement of 10·75 mm of test 1 and could demonstrate (along with the statement that both tests gave similar residual displacements) a similar behaviour of the unit before and after cracking.

3.2. Strain distribution across the depth of the riser

Figures 5 and 6 show the distribution of strain per load increment for an initially uncracked (test 1) and a cracked (test 2), section respectively. Strain was measured across the vertical symmetry line and at four different levels above the soffit of the riser (D11 = 40 mm, D10 = 110 mm, D9 = 165 mm and D8 = 235 mm).

The strain diagram in Figure 5 shows perfectly linear behaviour up to and including the load increment of 30 kN. This is compatible with the findings shown in Figure 4. The applied load was resisted by both concrete and reinforcement. Tension is gradually transferred to the reinforcement as the first cracks at the bottom of the unit appear, characterised by a nonlinear distribution of strain for load increments of 48, 60 and 72 kN. Equilibrium of the section is maintained by a gradual movement upwards of the neutral axis, reducing the area of section in compression. Figure 6 presents a similar account; this time the strain was distributed more smoothly. Once the cracks are developed, no sudden changes of strain are present during reloading. These cracks open wider following load increments, while strain readings reach higher values.

3.3. Strain measured at the reinforcement

Electrical resistance strain gauges were attached to the reinforcement as shown in Figure 3. SG1 was attached to the transverse reinforcement and SG2 to the longitudinal tension reinforcement of the riser. Their variation with load increments is shown in Figure 7. As expected, readings of strain gauge SG2 were found to be significantly higher than those of SG1, indicating that main bending took place in the longitudinal direction. Once again, the 30 kN and 48 kN loads were characterised by sudden strain jumps and possible local de-bonding. Maximum values recorded were: $SG1_{max} = 110\ \mu\varepsilon$ and $SG2_{max} = 2100\ \mu\varepsilon$ corresponding to magnitudes of stress of 22 N/mm^2 and 420 N/mm^2 respectively, assuming a modulus of elasticity for steel, $\boldsymbol{E}_{steel} = 200$ kN/mm^2 and a linear stress–strain relationship.

The strain curves for test 2 (cracked section) are a good deal smoother than those of test 1. Strain gauge SG1 showed no strain up to a load of 60 kN. It recorded a strain of 100 $\mu\varepsilon$ for

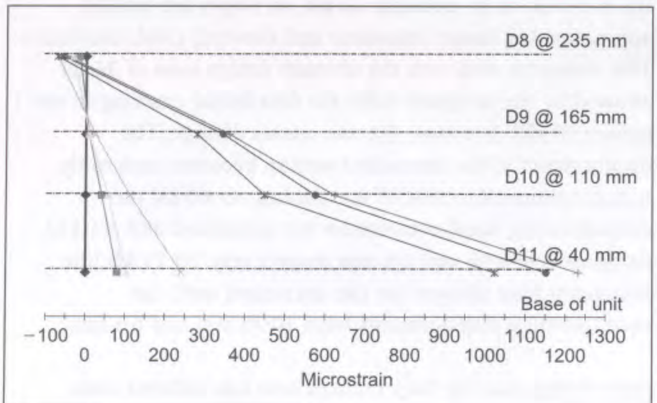

Figure 5. Terrace unit 1, test 1, strain distribution per load increment across the depth of the riser

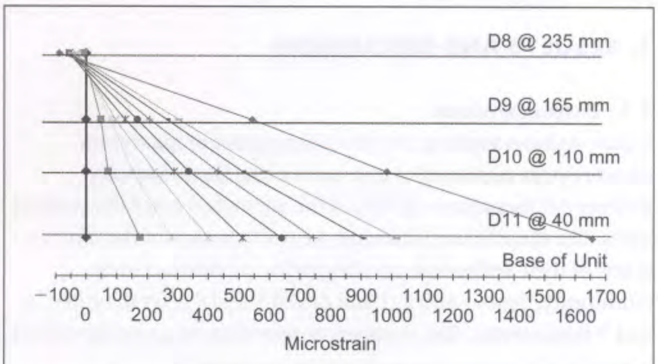

Figure 6. Terrace unit 1, test 2, strain distribution per load increment across the depth of the riser

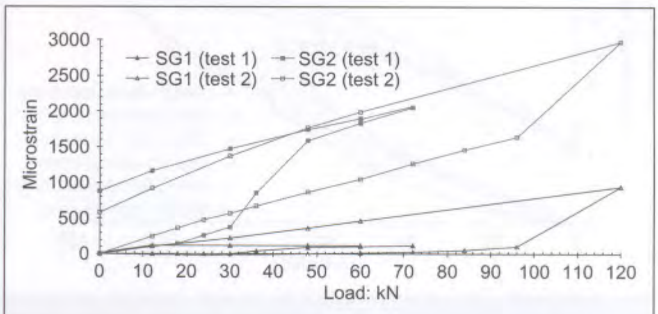

Figure 7. Terrace unit 1, tests 1 and 2, comparison between strains SG1 and SG2, loading–unloading

96 kN and a 'strain jump' to 900 $\mu\varepsilon$ (180 N/mm^2) for the final load of 120 kN. No residual strain was noticed after unloading, indicating that any cracks formed across the transverse reinforcement must have gradually closed during the unloading procedure. SG2, attached to the main tension reinforcement of the riser, showed a near linear behaviour reaching 1600 $\mu\varepsilon$ (320 N/mm^2) for 96 kN, before it finally reaches 3000 $\mu\varepsilon$ (600 N/mm^2, well beyond the yield stress of steel) for 120 kN.

3.4. Strain at SG1, D1, D3 and D4

Figure 8 shows very similar strain patterns for demec pairs D1, D3 and D4, as expected, confirming the validity and accuracy of the readings obtained. D1 reached a maximum and levelled at -500 $\mu\varepsilon$, D3 at -250 $\mu\varepsilon$ and D4 hovered around zero. There was a noticeable gradual reduction in lateral compressive strain from the extreme support regions to the symmetry line. This indicated an independent behaviour of the tread near the supports and a similar one to the riser near mid-span. That is, although the tread, as a structural section itself, developed compression and tension at top and bottom faces near the supports, the entire section was below the neutral axis of the riser (and therefore in tension) near the centre.

Based on Table 2, the total serviceability and ultimate state loads were 24·1 kN and 36·3 kN respectively. The longitudinal strain, measured at D7 and D6, turned tensile at 24 kN and 30 kN respectively (Figure 9). Also, the lateral strain at D4 turned tensile at 30 kN (Figure 8). This would indicate that when the unit is about to reach its allowable serviceability load, part of it does not obey classical RC theory as tension develops on the top surface. This has not been taken into account when designing the terrace units. The sensitivity of all three demec pairs was greatly reduced at test 2, resulting in strain readings very close to zero. It is envisaged that this was due to the development of a series of cracks outside the effective zone of the demec pairs. SG1, the strain gauge attached to the lateral bottom reinforcement of the tread, recorded tension in both tests. It is clear from Figure 8 that the top of the tread develops cracks at 12 kN (plain concrete), whereas the bottom develops its first cracks at 30 kN (composite action). It is important to remember that the top of the tread near the centre behaves in a different manner, with the whole (tread) section being in tension, following the behaviour of the riser. Yet, for cracking in the same direction, the bottom face will still crack under tension before the top face.

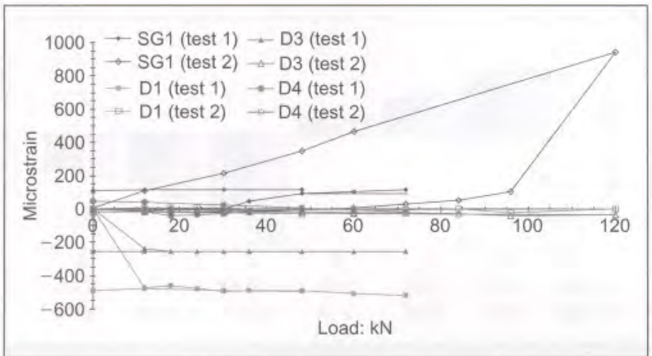

Figure 8. Terrace unit 1, tests 1 and 2, comparison between strains SG1, D1, D3, D4, loading–unloading

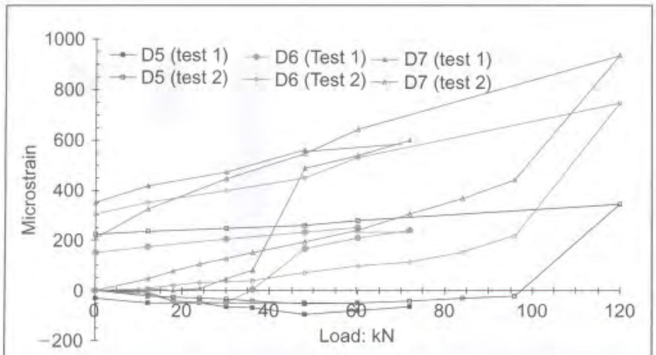

Figure 9. Terrace unit 1, tests 1 and 2, comparison between strains D5, D6, D7, loading–unloading

3.5. Strain at D5, D6 and D7

Demec point pairs D5, D6 and D7 were placed at mid-span measuring longitudinal strain as shown in Figure 3. Figure 9 shows the variation of strain with load as measured across these points. It is interesting to note that D5 has followed a compressive path, reaching strain of -100 $\mu\varepsilon$ at 48 kN and then somehow 'softening', to finish with -70 $\mu\varepsilon$ at the maximum load of 72 kN allowed for test 1.

In contrast, D6 and D7 have recorded considerably larger tensile strains. Tension in this region is somehow surprising, especially when this is developing along the span of the unit. It shows that the tread at the region identified between the riser and demec points D2 and D5 (Figure 3) follows the behaviour of the riser; that is, it is in tension and forms a 'trough'. The familiar pattern at 12 kN and 30 kN, discussed previously, is repeated here. There is, however, an enormous strain jump between load increments of 30 kN and 50 kN. This is more obvious from demec readings D6 and D7. As strain is recorded tensile, it confirms that concrete has yielded locally.

The loading and unloading paths during test 2 (fully cracked unit) were much smoother than the corresponding in test 1. D5 (Figure 9) showed a negative initial tendency with most of the readings appearing below the horizontal axis. This was also the norm of the corresponding demec pair for the uncracked unit in test 1. New cracks, or further opening of the existing cracks, appeared at 96 kN. The final strain values at 120 kN reached 350, 725 and 925 microstrain for D5, D6 and D7 respectively, whereas residual strains were 225, 300 and 200 microstrain for the same transducers.

3.6. Strains at SG2 and D11

Figure 10 shows a comparison between strains measured by strain gauge SG2, attached to the tension reinforcement of the riser and demec points D11, attached 40 mm above the bottom of the same riser. That is, both SG2 and D11 were at approximately the same level from the soffit of the riser. Initially, and up to the load of 18 kN, both strains showed good correlation, turning reasonably good up to 30 kN. Beyond the branded 30 kN load, however, SG2 produced considerably higher strain values than D11, although both strain paths were remarkably similar. This would indicate the beginning of failure for the surrounding concrete. Also, as new cracks

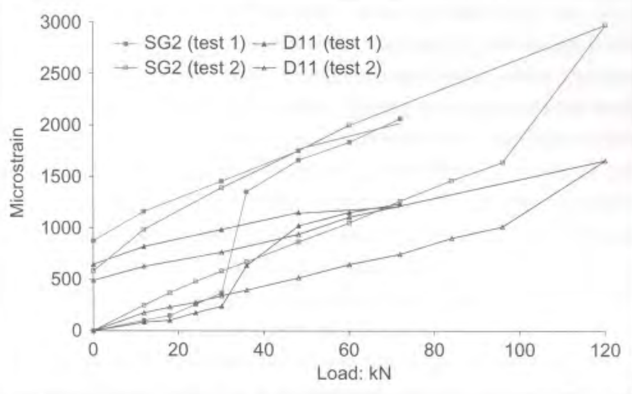

Figure 10. Terrace unit 1, tests 1 and 2, comparison between strains SG2 and D11, loading–unloading

develop in the vicinity, the crack captured by D11 has now ceased opening.

Maximum values of strain recorded for the final load of 72 kN were 2100 $\mu\varepsilon$ and 1200 $\mu\varepsilon$ for SG2 and D11 respectively.

There is as yet no standard test for directly determining the tensile strength of concrete; the latter can be taken as approximately equal to 1/10 of its compressive strength. Hence, in the absence of more scientifically based research, the modulus of elasticity of concrete in tension was taken as equal to 1/10 of that in compression. This allowed for the evaluation of the corresponding stresses for SG2 and D11, as 420 N/mm^2 and 3·6 N/mm^2 respectively. The latter should, however, be regarded with extreme caution.

The unloading procedure was carried out without surprises giving residual strain values of 880 $\mu\varepsilon$ and 640 $\mu\varepsilon$ for SG2 and D11 respectively.

The behaviour of the curve in test 2 was smoother and more linear compared with that of test 1. Although both strain paths are similar, SG2 produced higher strain values for reasons already explained above. Maximum strain was reached at 120 kN, with magnitudes equal to 3000 $\mu\varepsilon$ (600 N/mm^2) and 1650 $\mu\varepsilon$ (49·5 N/mm^2), for SG2 and D11 respectively. Both stress values were well beyond the yield stress values of the materials. Finally, unloading produced residual strains of 600 $\mu\varepsilon$ and 500 $\mu\varepsilon$ for SG2 and D11 respectively.

4. EVALUATION OF STIFFNESS

Static stiffness was estimated from the displacement–load graph in Figure 4. It was not possible to obtain a unique stiffness value as the slope of the F–δ curve changed each time a new crack appeared on the unit. The domain of F–δ curve of unit 1 was therefore divided into six convenient sub-domains and the 'best-fit' straight line was fitted in each one. The stiffness (slope) of the line was calculated based on the relationship

$$F = k\delta \quad \Rightarrow \quad k = F/\delta \tag{1}$$

The magnitude of these 'local stiffnesses' along with their difference—that is, uncracked minus cracked local stiffness (test 1-2)—are shown in Figures 11 and 12. It is apparent that when a new crack appeared on the unit, its stiffness was reduced. It is also clear that although stiffness values extracted from the uncracked unit were higher than those of the cracked, the former were not, in general, significantly higher. Also, the difference between the two reduced with increasing load, in some way highlighting the importance of steel reinforcement and the concept of ductility.

5. CONCLUSIONS

The following are evident from the incremental, static, loading–unloading tests, carried out on two precast concrete terrace units under laboratory conditions.

(*a*) The predominant mode of failure is the appearance of hair-like cracks at the soffit of the units and around the symmetry line (where bending stresses are maximum) and their gradual propagation upwards.

(*b*) The units are supported at the ends (under the riser) and propped along the front edge of the tread. Hence, they experience a combined bending and (to a lesser extent) torsional effect when loaded near the riser side. This has a knock-on effect on the deformed shape of the units, which is more complex than that assumed in their initial design. The numerical model developed in a separate part II paper[2] throws more light onto the problem.
It is correct to say that the assumptions and simplifications made at design stage did not have a detrimental effect on the static performance of the units. However, as the units behave more like plates (slabs) and less like beams, and if optimum design is to be achieved, it may be useful for the

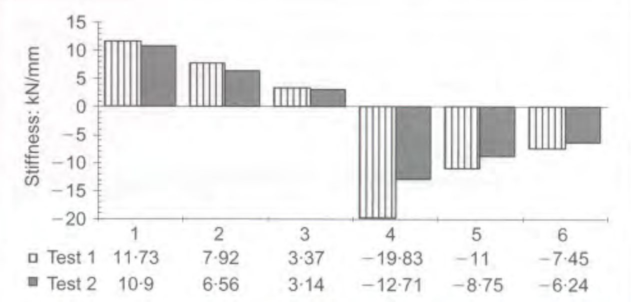

Figure 11. Terrace unit 1, tests 1 and 2, estimated stiffness values, loading–unloading

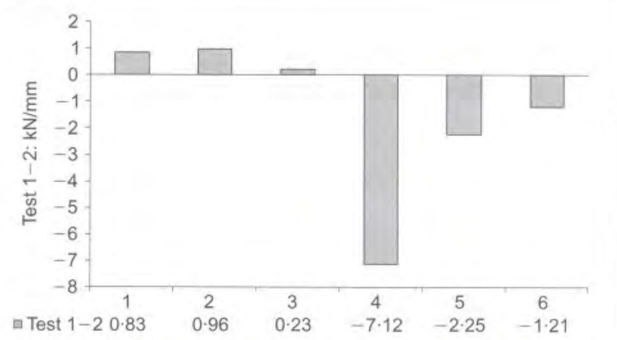

Figure 12. Terrace unit 1, tests 1 and 2, difference between uncracked and cracked local stiffness values, loading–unloading

latter to be approached from a more rigorous angle. The effect of the former on the dynamic performance of the units will be discussed in a different paper.

(*c*) As the corners of the tread tend to turn upwards (warping effect), separating themselves from the propping UB-section and the whole unit bends about two different axis (longitudinal and transverse), a 'trough' forms at the central region of the unit. The above leads to the conclusion that a 'plane of inflexion' (change from concavity to convexity or vice versa) is present. It has not been possible to define the locus of this plane accurately with the information obtained from the laboratory. This is reviewed in the part II paper,[2] given the results from a rigorous finite-element analysis.

(*d*) The maximum displacement of the uncracked unit was found to be higher than the corresponding one for the cracked unit for the same load value. This was because the maximum displacement measured for the cracked unit (test 2) was relative to the residual displacement inherited from test 1, and hence recorded lower. When, however, the permanent displacement from test 1 was added to the corresponding displacement obtained for test 2, the two displacements were found approximately equal, indicating similar irreversible behaviour of the unit before and after cracking.

(*e*) The strain distribution across the depth of the riser was found to be linear and remarkably similar in both tests. The linearity was more evident in test 2, as it is not accompanied by any substantial and sudden change in strain owing to the formation of additional cracks. When the tension zone was developing its first cracks, equilibrium was maintained by a shift of the neutral axis upwards, hence decreasing the sectional area in compression.

(*f*) Strain measured at the longitudinal reinforcement (SG2) is a good indicator of cracks appearing at the tension side of the unit. Strain measured at the lateral reinforcement (SG1) is approximately 21 times smaller than SG2. The 'strain loop' (loading–unloading) for the fully cracked section always enclosed that for the uncracked section.

(*g*) The static stiffnesses of the uncracked unit were found to be greater than that of the cracked unit, as expected, but their difference reduced gradually, with increasing load.

(*h*) The need for more research and reliable modulus of elasticity values for concrete in tension is highlighted.

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Concrete grandstands. Part II: numerical modelling

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This paper outlines the essential theoretical basis upon which a rigorous finite-element model comprising material non-linearities and failure criteria for both, concrete and steel reinforcement, is built. A numerical algorithm describing the analysis process, based on recent advances in numerical methods of reinforced concrete and a finite-element code were developed in parallel and summarised in a concise flowchart. It was concluded that the finite-element model captured successfully the non-linear flexural behaviour of the terrace units to failure, the formation of a 'bowl' at the centre, the lifting of the free tread ends and the rotation about a longitudinal axis. The results produced were rather sensitive to the modulus of elasticity assigned to concrete as well as the initial and, to a lesser extend, additional tangent moduli assigned to the reinforcement. The model was capable of predicting the introduction and propagation of flexural cracks formed around the midspan. More rigorous analytical and numerical work is under way, depicting both static and dynamic conditions, in an effort to establish suitable FE benchmarks, hence reducing uncertainties and increasing confidence of the performance of these structures during their working life.

NOTATION

E_c	modulus of elasticity of concrete
f'_c	uniaxial compressive cylindrical stress
f_{cu}	characteristic strength of concrete
f_t	tensile strength of concrete
I_1, I_2, I_3	first, second and third invariant of the stress deviator tensor
J_1, J_2, J_3	first, second and third invariant of the stress tensor
P_n	nth load step ($\forall n$: $n \in \aleph^+$)
s_{ij}	deviator stress
δ_{ij}	Kronecker delta
$\bar{\varepsilon}$	accumulated strain
ε_{ij}	general strain tensor
ε^e, ε^p	elastic and plastic strains respectively
σ_{ij}	general stress tensor
σ_m	hydrostatic mean stress
$\sigma_1, \sigma_2, \sigma_3$	principal stresses

1. INTRODUCTION: IDENTIFYING THE PROBLEM

Use of the finite-element (FE) method as a supplement to experiments and especially in situations where experimental work is either difficult to perform or cumbersome and expensive (e.g. reinforced concrete (RC) structures) has been increasing ever since the pioneering work of Ngo and Skordelis.[1] Extensive research has since resulted in significant advances in the area of constitutive concrete, leading to the development of a significant number of numerical models, partially listed in the reports of the American Society of Civil Engineers (ASCE) Committee on Finite Element Analysis of Reinforced Concrete Structures.[2]

The mechanical behaviour of RC as a composite material is not similar to that of its two basic constituents. Extensive research has, however, led to a few constitutive models for concrete based on the principles of continuum mechanics rather than the micro-mechanics of its molecular structure (crystallography). These models were based on the theory of elasticity following a linear or bilinear behaviour, or they incorporated a plasticity algorithm, or were capable of simulating plastic fracturing, or even a more general elasto-plastic behaviour.[3] This irregular behaviour of concrete as a material is attributed mainly to the following[4]

(*a*) the distinct non-linearity of its stress–strain path, especially in the near-peak domain, resulting from the development, growing and propagation of microcracks and the subsequent reduction in stiffness
(*b*) the softening tendency of concrete in the post-peak domain and the assemblage of these cracks in narrow bands
(*c*) the elastic stiffness dilapidation (decaying) caused by the successive opening and closing of cracks due to repeated loading–unloading
(*d*) the irrecoverable volume loss at high compressive loads resulting in an increase of Poisson's ratio.

Efforts will be directed in embracing most of the above in the numerical model presented below.

2. CONSTITUTIVE CONCRETE MODELS: THEORIES AND CRITERIA

Almost all classical theories of plasticity are based on five key concepts

(*a*) decomposition of strain into elastic and plastic parts
(*b*) yield criteria, determining the level at which yielding is initiated
(*c*) plastic flow rule, determining the direction of plastic strain

flow relative to x, y, z axes and the relationship between stress and plastic strain under multiaxial loading
(*d*) strain hardening rule, describing and controlling the changes the yield surface undergoes with progressive yielding, so that the various states of stress, and the way in which resistance to plastic flow increases with plastic straining, can be established
(*e*) the unloading condition, demonstrating the irreversible behaviour of the solid.

Failure criteria are introduced to assess the possibility of failure of the material. As plastic strains develop, the yield surface increases in size while maintaining its original shape (isotropic, plastic behaviour) or moves to a different location within the material (kinematic hardening), or both.

Although the behaviour of concrete under complex stress conditions has been under investigation for many years, there is as yet no universally accepted constitutive law. Concrete in tension has been modelled in several ways, the most successful being a linear elastic and strain softening material—that is, the principal stresses and their directions are computed initially for an uncracked concrete. If the maximum principal stress exceeds a limiting value, a crack is assumed to form in a plane orthogonal to this stress. After the first crack the behaviour of that region of concrete becomes orthotropic. Linear and exponential mathematical models have been used to describe the descending part of the stress–strain diagram.

Concrete in compression is treated in a similar manner. The strength parameters, the tensile and compressive strength of concrete, are used to define the initiation of fracture by means of a tension 'cut-off' condition in the principal stress plane. When the combination of principal stresses violates this condition, a crack is initiated.

Failure theories represent states of stress and/or strain at which concrete can no longer sustain one of the criteria such as, yielding, load-carrying capacity, crack initiation and deformation, leading to rather complex failure curves and failure surfaces. The amount of experimental data needed for the implementation of such failure envelopes has, however, been limited until now.

Using index notation and terminology, the state of stress at a point inside a concrete element can be completely defined by the stress tensor, $\boldsymbol{\sigma}_{ij}$, in a three-dimensional (3D) stress system, where each component of the stress tensor acts on a surface normal to the i-axis and in the direction of the j-axis. In general, the stress tensor can be decomposed into two parts: the hydrostatic (mean) stress, $\boldsymbol{\sigma}_{\mathrm{m}}$, involving only pure tension and compression and the deviator stress, $\boldsymbol{s}_{ij}$, involving only shear, as follows

$$\boldsymbol{\sigma}_{ij} = \boldsymbol{s}_{ij} + \boldsymbol{\sigma}_{\mathrm{m}}\boldsymbol{\delta}_{ij} \tag{1}$$

where δ_{ij} is the Kronecker delta: $\delta_{ij} = \begin{cases} 1, & \text{if: } i = j \\ 0, & \text{if: } i \neq j \end{cases}$ (e.g. $\delta_{12} = 0$, but $\delta_{33} = 1$).

The condition for failure due to a multiaxial stress state is actually based on a model developed by Argyris,[5] who suggested a three-parameter criterion involving both stress invariants. Willam and Warnke[6] expanded on the Argyris model by adding two additional parameters, degrees of freedom (DOF), for describing meridian sections (surface generating curves) so that the failure surface model can be applied to low as well as high compression regions. Hence, they introduced a five-parameter criterion in which the tensile and compressive meridians were expressed by

$$\sqrt{\boldsymbol{I}_{2,\mathrm{t}}} = A_{01} + A_{11}\boldsymbol{J}_1 + A_{22}\left(\frac{\boldsymbol{J}_1{}^2}{\boldsymbol{f}_{\mathrm{c}}{}'}\right) \quad \text{tensile meridian } (\theta_{\mathrm{c}} = 0) \tag{2}$$

$$\sqrt{\boldsymbol{I}_{2,\mathrm{c}}} = B_{01} + B_{11}\boldsymbol{J}_1 + B_{22}\left(\frac{\boldsymbol{J}_1{}^2}{\boldsymbol{f}_{\mathrm{c}}{}'}\right) \quad \text{compressive meridian } (\theta_{\mathrm{c}} = 60°) \tag{3}$$

where A_{01}, A_{11}, A_{22}, B_{01}, B_{11}, B_{22} are constants, $\boldsymbol{f}_{\mathrm{c}}'$ is the uniaxial compressive cylindrical stress (straight meridians), $\boldsymbol{J}_1 = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2)$, the first invariant of the stress tensor and $\boldsymbol{I}_2 = \frac{1}{2}(\boldsymbol{s}_1 + \boldsymbol{s}_2 + \boldsymbol{s}_3)$ the second invariant of the stress deviator tensor

They were able to develop an expression for the failure curve by modelling it as part of an ellipse in the deviatoric plane. The five-parameter model was validated by utilising experimental data from Launay and Gachon.[7] It is characterised by a smooth surface and produces the main features of the triaxial failure surface of concrete. This is the criterion adopted by Ansys.[8]

In comparison, little work has been done in treating RC structures as 3D solids mainly because of the relative lack of knowledge of the concrete as a material under three-dimensional stress states. The most successful theory is probably attributed to Selby and Vecchio.[9] Their FE model used the secant stiffness (as opposed to tangent modulus) thus allowing for stability of the non-symmetric matrices developed but was only effective for short-term monotonic loads. Han and Chen[10] experimented with elasto-plastic constitutive models for concrete under triaxial states of stress. They used a hardening parameter in their non-uniform hardening plasticity model to simulate inelastic behaviour of concrete including brittle failure in tension, ductile behaviour in compression and volumetric dilatation under compressive loading. Their model could fit a wide range of experimental data, treating them as parameters such as shape factor, plastic modulus, modification factor and so on.

An alternative material model based on the theory of plasticity is that permitting dependence of strain on the loading history of the material. Hence, based on small strains theory, for multi-axial stress states and as irreversible material behaviour governs, the strain tensor can, in general, be decomposed into elastic and plastic strain increments such as

$$\mathrm{d}\boldsymbol{\varepsilon}_{ij} = \mathrm{d}\boldsymbol{\varepsilon}_{ij}^{\mathrm{e}} + \mathrm{d}\boldsymbol{\varepsilon}_{ij}^{\mathrm{p}} \tag{4}$$

3. STEEL REINFORCEMENT: ELASTO-PLASTIC BEHAVIOUR

For metals, including steel, the von Mises yield criterion with its associated flow rule and work (isotropic) hardening is adopted. For the case of reinforcement undergoing tension or compression, uniaxial conditions were assumed and a bilinear isotropic hardening approach was chosen to simulate the behaviour of steel. The material is assumed to undergo yielding when the equivalent stress reaches the yield stress. At the same time the corresponding yield surface depends solely upon the amount of plastic work done. It was therefore necessary to define and input the yield stress and a tangent modulus (gradient) after yielding, plus the modulus of elasticity and Poisson's ratio.

The final solution was obtained by utilising the linear solution, modified with an incremental and iterative approach. In general

$$\boldsymbol{\varepsilon}_n = \boldsymbol{\varepsilon}_{n(\mathrm{ela})} + \Delta\boldsymbol{\varepsilon}_{n(\mathrm{pla})} + \boldsymbol{\varepsilon}_{n-1(\mathrm{pla})} \tag{5}$$

where $\boldsymbol{\varepsilon}_n$ is the total strain at the nth iteration, $\boldsymbol{\varepsilon}_{n(\mathrm{ela})}$ is the elastic strain for the same iteration, $\Delta\boldsymbol{\varepsilon}_{n(\mathrm{pla})}$ is the additional plastic strain obtained from the same iteration and $\boldsymbol{\varepsilon}_{n-1(\mathrm{pla})}$ is the total and previously obtained plastic strain.

Convergence is achieved when Equation 6, is satisfied[11]

$$(\Delta\boldsymbol{\varepsilon}_{n(\mathrm{pla})}/\boldsymbol{\varepsilon}_{n(\mathrm{ela})}) < 0{\cdot}01 \tag{6}$$

This means that very little additional plastic strain is now accumulating and therefore the theoretical curve, represented by two straight lines (bilinear approach), is very close to the actual one.

4. NUMERICAL MODELLING

A step-by-step FE analysis (FEA) algorithm is developed in which provision is made for cracking and crushing of concrete (brittle failure) and the elasto-plactic behaviour of both materials, under static incremental loading conditions. A summary of the algorithm is shown in Figure 1.

4.1. Cracking and crushing of concrete

Based on the statements in Section 2, and assuming $\boldsymbol{\sigma}_1 \geqslant \boldsymbol{\sigma}_2 \geqslant \boldsymbol{\sigma}_3$, the failure condition of concrete could be divided into four discrete domains

(*a*) when $0 \geqslant \boldsymbol{\sigma}_1 \geqslant \boldsymbol{\sigma}_2 \geqslant \boldsymbol{\sigma}_3$, (i.e. compressive–compressive–compressive), crushing occurs
(*b*) when $\boldsymbol{\sigma}_1 \geqslant 0 \geqslant \boldsymbol{\sigma}_2 \geqslant \boldsymbol{\sigma}_3$, (i.e. tensile–compressive–compressive), cracking occurs
(*c*) when $\boldsymbol{\sigma}_1 \geqslant \boldsymbol{\sigma}_2 \geqslant 0 \geqslant \boldsymbol{\sigma}_3$, (i.e. tensile–tensile–compressive), cracking occurs
(*d*) when $\boldsymbol{\sigma}_1 \geqslant \boldsymbol{\sigma}_2 \geqslant \boldsymbol{\sigma}_3 \geqslant 0$, (i.e. tensile–tensile–tensile), cracking occurs.

As mentioned earlier, five input strength parameters are needed to define the failure surface, as well as a hydrostatic stress state. These are summarised in Table 1. The failure surface can be specified by a minimum of two constants, $\boldsymbol{f}_t$ and $\boldsymbol{f}_c$, whereas the other three constants default to specific values if the hydrostatic stress component, $\boldsymbol{\sigma}_m$, is low (all five parameters are needed if $\boldsymbol{\sigma}_m$ is high).[6]

4.2. Shear transfer

The discrete representation of reinforcement within the framework of the FE method is based on modelling the concrete and the reinforcing bars as different elements.

Ansys[8] recommends a 3D, eight-node, solid, isoparametric element (Solid65), with three translational DOF per node to simulate the nonlinear response of brittle materials such as concrete. For cracking in the tension zone the element includes a smeared crack analogy allowing cracks to be shown in the deformed shape. After the formation of the first crack, stresses tangential to the crack face may cause a second or third crack to develop and so on. The amount of shear transfer can be adjusted in the concrete material data table. This allows additional concrete material data such as tensile and compressive strengths and 'shear transfer coefficients' to be input in the program. The latter range from 0, representing a perfectly smooth crack with total loss of shear transfer, to 1, representing a perfectly rough crack with no loss of shear transfer. For crushing in the compression zone, it follows a typical plasticity law—that is, once the section has crushed any further application of load develops increasing strains at constant stress.

The element behaves in a linear elastic manner, if the applied tensile or compressive stress is less than the tensile or compressive strength of the material. If one of the applied principal stresses exceeds the tensile or compressive strength, however, then cracking or crushing of the element starts. Accordingly, cracked or crushed regions are formed perpendicular to the relevant principal stress direction. In the numerical routines the crack formation is achieved by the modification of the stress–strain relationships of the element, hence introducing a plane of weakness in the required principal stress direction.

It is stressed here that the numerical analysis routines incorporated in the program dictate that cracked or crushed regions (and single cracks) are formed perpendicular to the direction of the applied principal stress, which has just exceeded the corresponding tensile or compressive strength of the material. Hence, in a typical flexural test and at regions near the mid-span, cracks should appear vertical (see Figure 10 later) whereas at regions near the supports they should be inclined at an angle of approximately 45° to the horizontal.

4.3. The problem of smeared reinforcement

The reinforcement could be modelled as an additional smeared stiffness, distributed through the centroid of the element in 3D Cartesian system orientation. Up to three different rebar specifications may be defined this way. They can resist tension and compression but surprisingly, no shear. The problem does not seem to be fully alleviated with the usual remedies such as the introduction of discrete tie-strut (Link8), or beam (Beam4) elements connected to the solid elements (Figure 2). This is because the beam elements would allow the reinforcement to develop shear stresses but, as they are primarily linear elements, they would not go beyond yielding and therefore no plastic deformation of the reinforcement is possible. The link

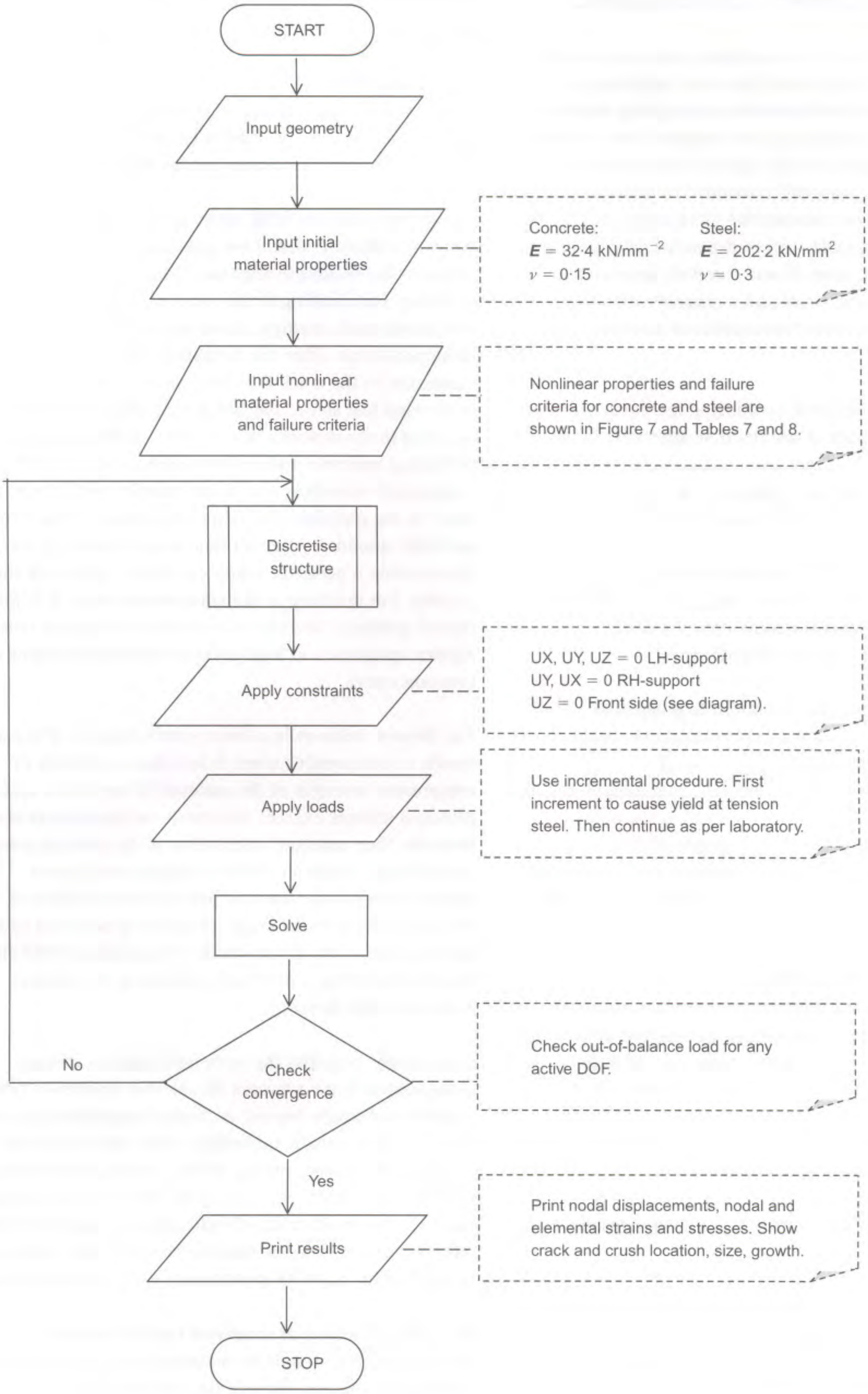

Figure 1. Symbolic flowchart representation of the algorithm describing the analysis process

elements, on the other hand, would allow elasto-plastic response of the reinforcement to be introduced in the RC simulation but, like the smeared reinforcement of the solid elements, no shear stress stiffness modelling is possible.

This problem can be averted by first considering the mechanism of shear transfer in a cracked concrete beam. This states that the applied shear stresses are resisted by the combined action of shear in the uncracked compression zone (approximately 30%), plus the contribution owing to aggregate interlock (approximately 45%), plus the dowel action of the longitudinal reinforcement (25%).[12] Taylor[12] demonstrated

Parameter	Description
f_t	Ultimate uniaxial tensile strength
f_c	Ultimate uniaxial compressive strength
f_{cb}	Ultimate biaxial compressive strength
σ_H	Hydrostatic stress (ambient)
f_1	Ultimate compressive strength for the state of biaxial compression superimposed on σ_H
f_2	Ultimate compressive strength for the state of uniaxial compression superimposed on σ_H

Table 1. Parameters used to define a failure surface (failure criterion by Willam and Warnke[6])

handle such behaviour. This is because even zero stiffness at the unstable region (top of the curve, where the stiffness matrix changes its sign from positive to negative), possesses a problem for the N–R method. The latter becomes singular, inputted constraining equations become inadequate, the technique predicts an unbounded displacement increment and the model is declared unstable, often preventing further solution.

More recent advances promise solvers; they can offer sophisticated solution techniques such as Riks'[13] and Crisfield's[14,15] arc-length method. These solvers are incorporated in recent Ansys codes but they are bounded by restrictions such as only being suitable for certain elements and when the loading is strictly proportional; that is, where the load magnitudes are governed by a single scalar parameter. When the above is not the case, they do not obtain good results.

Attributing strain-softening characteristics to the post-peak behaviour of concrete seems, however, to be contradicting its brittle nature, especially when past studies have shown that strain softening is merely attributed to the interaction between specimen and loading platens of the apparatus.[16–18] In other words, if edge effects were eliminated, then concrete should be characterised by a complete and immediate loss of load-carrying capacity, as soon as its peak strength is reached. Hence, the well-known descending part of every laboratory-obtained concrete stress–strain curve is questionable, to say the least.

A series of laboratory tests, in a different research area, are currently being carried out at Coventry University on concrete beams and slabs with synthetic reinforcement. Although it is still too early for any firm conclusions it is, however, worth mentioning that the phenomenon of strain softening is absent, indicating that the softening effects may be attributed to the ductile behaviour of steel reinforcement and the composite action of the two materials.

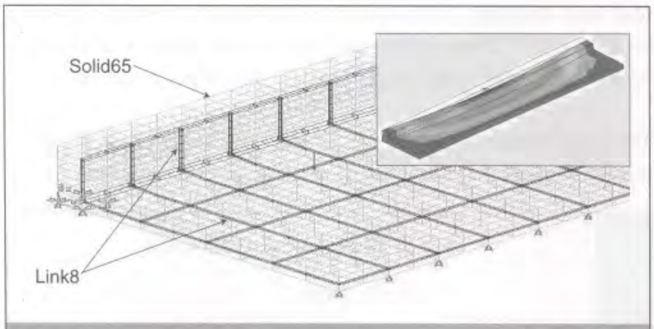

Figure 2. An FE model of a single terrace unit, steel elements are shown exaggerated for clarity. Inset: deformed shape of the unit, dark regions at the edges are virtually undeformed

that, as the applied shear force is increased, the dowel action is the first to reach its capacity after which a proportionally large shear is transferred to aggregate interlock.

The contribution the main steel (here modelled with Link8-elements) would therefore provide to shear resistance could be credited (passed) to the surrounding concrete and can even be specified for either both, open and closed cracks or one case only, as shown in Table 2. When cracking is imminent and the solution is converging to the cracked state, the modulus directly normal to the crack is significantly reduced and when the crack is open it is set to near zero. At this stage, the stiffness normal to the crack face will also be zero.

4.4. Softening of concrete

Concrete, unlike steel, shows a post-yield, strain-softening behaviour, obtained from routine tests on specimens such as cubes, cylinders, prisms and so on. This means that its stress–strain relationship follows a downward path after yielding. The traditional nonlinear solutions such as the Newton–Raphson (N–R), or modified Newton–Raphson (mN–R) techniques cannot

4.5. Concrete–steel interaction

The transfer of forces across the interface between concrete and steel reinforcement by bond is of fundamental importance, as

Shear transfer contribution as defined by Taylor[12]: %	For closed cracks: %	For open cracks: %	Ansys input. Closed cracks: shear transfer coefficient	Ansys input. Open cracks: shear transfer coefficient
Dowel action: 25	25	25	0·25*	0·25*
Aggregate interlock: 45	45	<45	0·40 + 0·25*	0
Compression zone: 30	30	30	0·3	n/a
			Total: 0·95	Total: 0·25

*Coefficient 0·25 (contribution of re-bars to shear transfer) is carried over.

Table: 2. Shear transfer coefficients and percentage of shear transfer attributed to concrete flexural and other actions can cause the steel to slip through the concrete in a direction parallel to the bars. The interaction between concrete and steel is described by the assumption of a perfect bond. The latter is rather compatible with the smeared crack model from the point of view that no detailed description of the local effects is necessary. Also, as bond failure is a long process, the concrete material surrounding the reinforcement would probably have started departing the steel before the maximum bond stress is reached and any attempt to restrict these stresses to bond strength would be flawed. In any case, the detailed representation of the bond mechanism between steel and concrete is outside the scope of a macro/meso-scale mechanics model such as the present one.

Nevertheless, Link8 elements were embedded within the solid mesh by sharing common nodes with the Solid65 elements. In this case the unaided discrete representation of the reinforcement by one-dimensional, Link8 elements, connected to 3D, Solid65 mesh, was not made to account for possible displacement of the reinforcement relative to the surrounding concrete.

4.6. Computer simulation of laboratory tests

A series of comprehensive laboratory tests have been carried out elsewhere (Part I).[19] The aim was twofold. First, to investigate the behaviour of a family of RC terrace units supported at three positions and undergoing static, incremental loading. Second, to estimate the uncracked and fully cracked stiffness of the units. Two tests per unit were carried out for the latter aim. Test 1 assumed the section uncracked as it was transported from the factory and test 2 considered the same section, this time fully cracked, as received from test 1.

The geometry of the L-section terrace units and their design features are presented in Figure 3 and Tables 1, 2 and 3 of the companion part I paper.[19]

Figure 3 of the current paper shows the behaviour of typical concrete cylinders in uniaxial compression in accordance with the BS 1881,[20] its initial part being approximated by the 'best-fit' straight line. Softening, in this case, is regarded as a 'post-failure' phenomenon. The slope of the best-fit line was estimated to be 30·04 kN/mm^2.

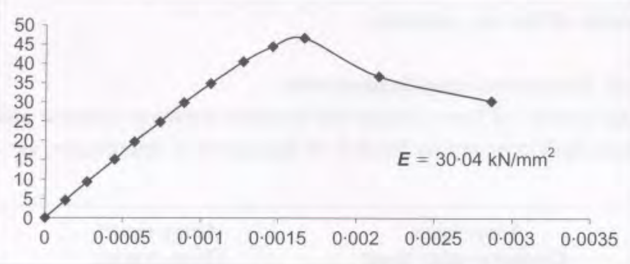

Figure 3. Stress–strain behaviour of grade C45 concrete in compression and estimation of its modulus of elasticity

Clearly, the correlation of test and numerical data depends a great deal on the assignment of accurate linear and nonlinear material properties, as appropriate. Hence, Young's modulus of concrete was related to its compressive strength by[21]

$$\boldsymbol{E}_{\mathrm{c}} = 9100\ (\boldsymbol{f}_{\mathrm{cu}})^{1/3} \tag{7}$$

In this case Hughes'[21] relationship yields a Young's modulus of 32·367 kN/mm^2 which is just 7·6% out of the average secant (static) modulus of elasticity value of 30·04 kN/mm^2 recorded in the laboratory. Also, it is less than 12·3% out of the average value of 28·80 kN/mm^2 obtained from ultrasonic laboratory tests and less than 3·4% out of 33·5 kN/mm^2, the value given by BS 8110.[22] These values are tabulated in Table 3. The value of 30·00 kN/mm^2 was adopted for FE modelling purposes.

The same British Standard for RC provides an estimate for its tensile strength based on its known compressive strength by

$$\begin{aligned} \boldsymbol{f}_{\mathrm{t}} &= 0{\cdot}36\ (\boldsymbol{f}_{\mathrm{cu}})^{1/2} = 0{\cdot}36\left(\sqrt{45}\right) \\ &= 2{\cdot}42\ \ \mathrm{Nmm}^{-2} \end{aligned} \tag{8}$$

The initial input properties for RC are shown in Table 4, whereas Table 5 was prepared to provide specific failure criteria for the concrete model.

Following the theory outlined in Section 3, the bilinear behaviour of deformed high-yield re-bars under tensile incremental loading was depicted in routine laboratory tests and is presented in Figure 4. The initial slope of the curve was taken as the elastic modulus of the material (198 kN/mm^2). Up to this point in the FE model the steel reinforcement is set to behave in a linear elastic manner. Plasticity is then introduced at a specified 0·2% proof stress (525 N/mm^2). The curve continues along the second slope defined by a tangent modulus (1·06 kN/mm^2). Failure occurs when the calculated value of stress reaches the ultimate value (σ_u= 660 N/mm^2) of the material. Failure criteria for steel in the form of stress and strain values are tabulated in Table 6.

The effect of self-weight of the terrace units was not taken into consideration when the laboratory tests were carried out–that is, for zero applied load the displacement was set to zero, although microcracks tend to develop in concrete at a very early stage, mainly owing to shrinkage. The same conditions for the theoretical model ensued.

4.7. Finite-element solution procedure

An essential feature of the nonliner solution strategy is the incremental application of load and the iterative procedure per load increment, hence the update of the stiffness matrix. In rate-independent material models these load steps take the role of the time steps. The computation of the state of strain and

E_{Hughes}	E_{static}	$E_{\mathrm{ultrasonic}}$	E_{BS8110}	E_{FEA}
32·367	30·04	28·80	33·5	30·00

Table 3. Moduli of elasticity values for concrete in kN/mm^2

Concrete		Steel re-bars	
E_c	30 kNmm^{-2}	E_s	198 kN/mm^{-2}
f_{cu}	45 Nmm^{-2}	f_y	– N/mm^{-2}
f_t	2·42 Nmm^{-2}	0·2% proof stress	525 N/mm^{-2}
ν_{con}	0·15	ν_{steel}	0·3

Table 4. Initial material properties as derived from design (see Table 3, Part I: experimental investigation[19]) and routine laboratory tests

Failure criteria for concrete: stress (N/mm^2)								
σ_x (tens)	σ_x (comp)	σ_y (tens)	σ_y (comp)	σ_z (tens)	σ_z (comp)	σ_{xy}	σ_{yz}	σ_{zx}
2·42	−45	2·42	−45	2·42	−45	0·45	0·45	0·45
Failure criteria for concrete: strain								
ε_x (tens)	ε_x (comp)	ε_y (tens)	ε_y (comp)	ε_z (tens)	ε_z (comp)	ε_{xy}	ε_{yz}	ε_{zx}
0·0001	−0·00175	0·0001	−0·00175	0·0001	−0·00175	—	—	—

Table 5. Failure criteria for concrete as inputted in the FE model

stress at a particular integration point (an interface the system uses to communicate with another application) is followed by a check against material failure. If failure is detected, then the type of failure (cracking–crushing), is established. Ansys[8] uses the smeared crack model for the representation of cracks with some advantages against discrete models, such as arbitrary directioning and no need for mesh adaptability and refinement. A drawback can be their disability to permit reliable description of the material behaviour in the vicinity of the crack and therefore only suited in macro-scale mechanics applications. Most of the smeared crack models reported in relevant literature are based on the theory of elasticity and only a small number of studies such as the one by Feenstra and de Borst[23] have reported plasticity models.

As plasticity is path dependent, it necessitates that, in addition to multiple iterations per load substep, the loads be applied slowly, in increments, with the presence of convergence tests in each substep, in order to characterise and model the actual (laboratory) load history. This is achieved by the NSUBST (number of substeps) command, defining the number of substeps to be taken within a load step. Ansys recommends a practical rule for load increment sizes, such as the corresponding additional plastic strain does not exceed the order of magnitude of the elastic strain. This can be achieved by applying additional load increments not larger than the load in step one, scaled by the ratio $E_T/E > 0{\cdot}05$. Such as

$$P_{n+1} = \frac{E_T}{E} P_n \qquad \forall n \in N \tag{9}$$

where
P_{n+1} is the $(n+1)$th load step, E is the elastic slope, E_T is the plastic slope and P_n is the (n)th load step.

Load substep one was chosen so as to produce maximum stresses approximately equal to the yield stress of the material. The yield stress was taken as 525 N/mm^2 from routine laboratory tests (Figure 4) and the corresponding load of 24·5 kN was noted. Also, the load to initiate yield was selected by performing a linear elastic analysis with a unit load and by restricting the stresses to the yield stress of the material. This was found to be approximately equal to 23·7 kN. The same design load at serviceability state conditions was 24 kN (Table 2, Part I).[19] In an effort to minimise uncertainties the number and size of successive load substeps were made to approximate the load history in the laboratory.

5. RESULTS AND DISCUSSION

5.1. Load–deflection

The experimental load-deflection response of unit 1 is reproduced in Figure 5, plotted along with the FE results. Test 1 represents the uncracked unit and test 2 the same unit cracked. It is clear from the path of the FE curve that the response of the model is linear until the first crack has formed at approximately 24 kN. This compares very well with the experimental findings. In test 2 (cracked unit) cracks inherited from test 1 have smoothened the overall behaviour of the unit.

The ultimate measured and predicted loads reached and the corresponding mid-span deflections are shown in Table 7. A large number of FE models depicting concrete behaviour predict deflections that are noticeably lower than the measured ones. This may be attributed, to a certain extent, to the plastic properties of the reinforcement, which can be such that converged solutions cannot be achieved beyond a certain load step, corresponding to a particular deflection value.

5.2. Strain variation

Figure 6 shows the measured and predicted strain variation at the lateral (SG1) and longitudinal (SG2) reinforcement at mid-span (Figure 6, Part I)[19] and Table 8 shows the ultimate strain values for both measured and predicted strains. All strain values are below the ultimate value of 3330 $\mu\varepsilon$. The strain

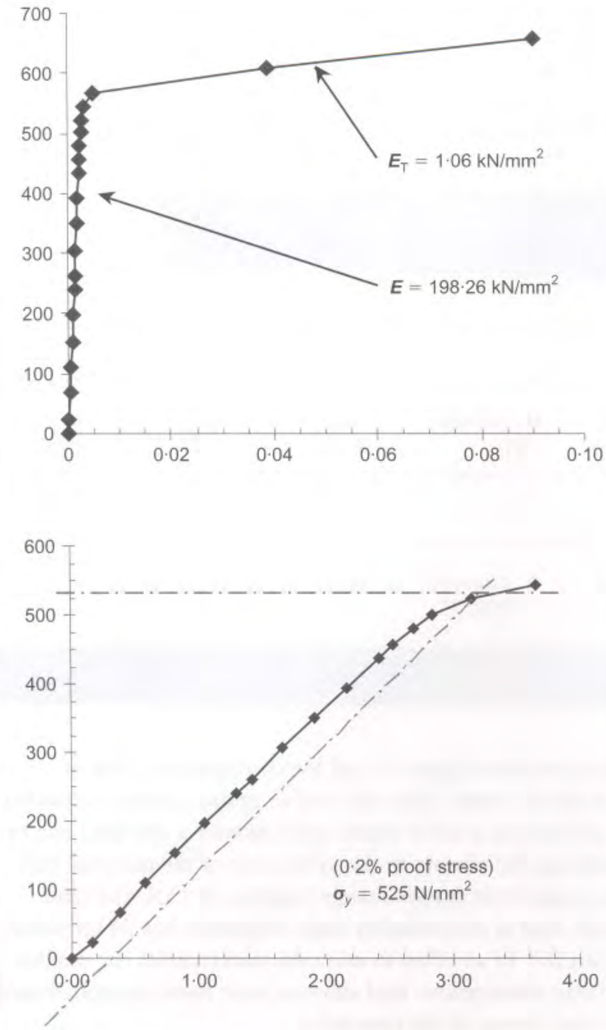

Figure 4. Stress–strain behaviour of type 2 high-yield steel reinforcement and determination of its modulus of elasticity and 'tangent' modulus

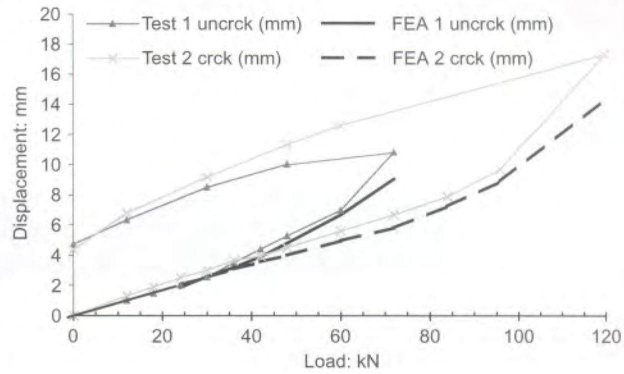

Figure 5. Comparison between measured and predicted displacements

	Test 1 (uncracked unit)	Test 2 (cracked unit)
Measured (**W**, δ): (kN, mm)	(72, 10·7)	(120, 17·2)
Predicted (**W**, δ: (kN, mm)	(73, 9·08)	(126, 14·80)

Table 7. Measured and predicted ultimate values of load and mid-span displacement

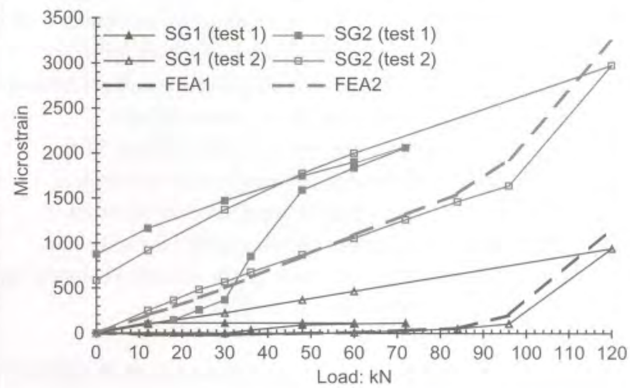

Figure 6. Comparison between measured and predicted strains developing at the reinforcement

variation for the uncracked section is not depicted for reasons discussed earlier (see Section 4.4).

5.3. Flexural strain distribution

The distribution of strain across the depth of the riser at mid-span is shown in Figures 7 and 8. In order to facilitate a direct comparison, efforts were concentrated in harmonising, where possible, the load increments followed in the laboratory with those chosen by Ansys in the nonlinear analysis. The resemblance of the results obtained is very satisfactory (compare Figures 4 and 5, Part I).[19]

In addition, the experimental crack formation at mid-span, Figure 9, compares well with the crack formation predicted by the FE model, shown in Figure 10. Cracking and crushing in Ansys are displayed by small circles and octahedra at locations

Failure criteria for steel: stress (N/mm^2)								
σ_x (tens)	σ_x (comp)	σ_y (tens)	σ_y (comp)	σ_z (tens)	σ_z (comp)	σ_{xy}	σ_{yz}	σ_{zx}
660	−660	660	−660	660	−660	—	—	—
Failure criteria for steel: strain								
ε_x (tens)	ε_x (comp)	ε_y (tens)	ε_y (comp)	ε_z (tens)	ε_z (comp)	ε_{xy}	ε_{yz}	ε_{zx}
0. 09	−0·09	0. 09	−0·09	0·09	−-0·09	—	—	—

Table 6. Failure criteria for steel as inputted in the FE model

Lateral reinforcement		Longitudinal reinforcement	
Test No:	(kN, $\mu\varepsilon$)	Test No:	(kN, $\mu\varepsilon$)
1	(72, 114)	1	(72, 2058)
2	(120, 940)	2	(120, 2974)
FEA1	(126, 1167)	FEA2	(126, 3238)

Table 8. Measured and predicted ultimate values of load and mid-span strains

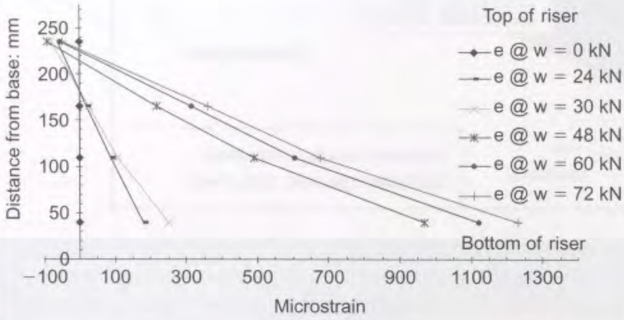

Figure 7. Unit 1, test 1. Predicted stress distribution across the depth of the riser

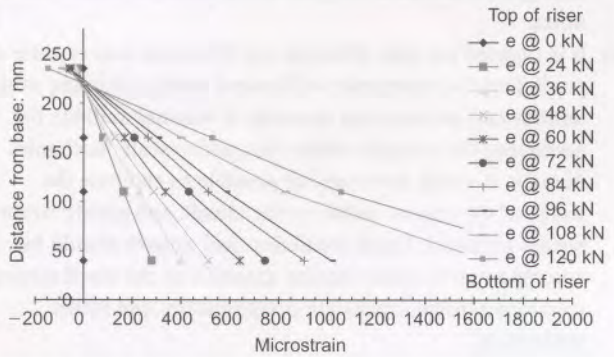

Figure 8. Unit 1, test 2, predicted stress distribution across the depth of the riser

Figure 9. Front view detail of the riser at mid-span, showing cracks developed in the laboratory

where concrete has cracked, or crushed respectively. If the crack has opened and then closed it is marked with 'x' through the circle. There is cracking evident in Figure 10, confirming that tension at the lower parts of the riser has been passed to the reinforcement.

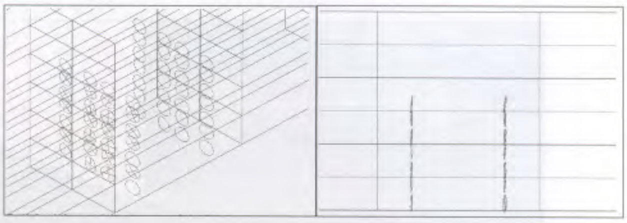

Figure 10. Isometric and front elevation (translucent view) details of the riser at mid-span showing predicted cracks by Ansys

5.4. Supplementary strain results

Finally, Figure 11 shows a comparison between measured and predicted strain readings for maximum load (72 kN) at specific positions on the surface of the terrace unit. It was shown, both experimentally and theoretically, that there was a gradual reduction in lateral compressive strain (and hence an increase in tensile strain) from the extreme support regions to the centre of the unit. This indicated an independent behaviour of the tread at the extremes and a similar behaviour to that of the riser near the middle. It also indicated the tendency of the unit to 'sink' in the middle and rotate about a horizontal longitudinal axis. In fact, the deformed FE model predicted a similar 'bowl' being deployed around mid-span and displayed torsional evidence of the unit about its longitudinal axis (inset, Figure 2). The tendency of the unit in the laboratory to leave the continuous support at the front and lift its corners has also been 'dramatised' by the computer, as nodal reactions were found negative (Figure 11).

Ansys provides a series of dedicated Contact elements that can model opening and closing, or sliding (friction) between two surfaces. These (nonlinear) elements would suit a condition such as lifting of the unit near the corners of the tread. If the emphasis of the analysis were at both the unit and the supporting medium, then best modelling practice would probably dictate the use of these elements. As, however, this is typical plate (slab) behaviour and as lifting can be clearly seen in the deformed shape of the FE model, it was decided that any more complex simulation of the interface between the UB-section, used as support, and the concrete unit should be unnecessary.

6. CONCLUSIONS

An accurate FE model of an RC terrace unit was developed in Ansys 7.0, by employing the dedicated concrete Solid65 and the Link8 elements and data obtained from a parallel laboratory investigation. The general elasto-plastic constitutive approach with the cracking and crushing options has captured successfully the nonlinear flexural behaviour of this composite unit to failure. The dedicated Solid65, 3D-element has been developed specifically for RC. No other element, including the family of powerful Shell (3D) elements, would be able to match the capabilities of the above and especially the modelling and prediction of cracks.

In general, 2D and Shell elements are best suited for 'membrane' or 'thin-walled' structures where the variation of stresses along the third dimension is either negligible, or of little interest. This is not the case with the (asymmetric cross-

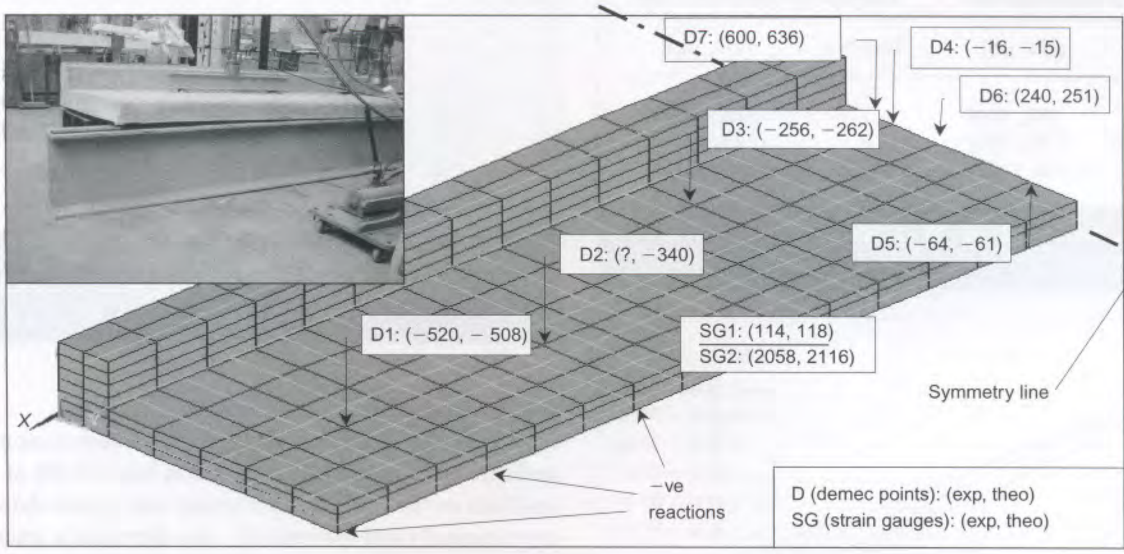

Figure 11. Unit 1, test 1, comparison between experimental and theoretical strains for maximum load of 72 kN at certain positions on the unit. D1, D2, D3, D4 are strains in the lateral, y-direction; D5, D6, D7 are strains in the longitudinal, x-direction. SG1 and SG2 are lateral and longitudinal strains developing on the reinforcement. Negative reactions, predicted by the FE model, are also shown in the inset, as lifting at the corners. All strains in $\mu\varepsilon$

section) terrace unit as its behaviour (bending and torsional effects, 'lifting' at the edges and 'sinking' at the centre) has shown. The use of solid elements will be reviewed at the next stage, when the whole grandstand is to be modelled.

The following conclusions can be made.

(a) The mode of failure predicted by the numerical model was of a flexural nature owing to increasing plastic strains developing in the tension zone (reinforcement). It was consistent with the experimental response.
(b) The FE model depicted accurately the formation of a 'bowl' at the centre of the unit and a 'region of inflexion' (change of sign of the bending moment values) surrounding it. The tendency of the tread to lift at the free ends (typical slab performance, noted during the experimental investigation) was also depicted by the numerical model, as corresponding node reactions were predicted negative. Finally, rotation of the units about a longitudinal axis was also captured.
(c) The results from the FE model were found to be rather sensitive to the modulus of elasticity assigned to the concrete as well as the initial and, to a lesser extent, additional tangent moduli assigned to the reinforcement. The various parameters controlling the nonlinear performance of the model are, however, numerous and mainly depend on the materials, geometry and the numerical techniques employed.
(d) It was found that in order to control the position of the reinforcement with accuracy and therefore achieve better results, it is necessary to simulate the latter in a discrete rather than smeared manner. Also, the inability of smeared reinforcement to transfer shear stresses discretely is still to be addressed. Hence, the development of dedicated elements is recommended. The Solid65 elements and the numerical models based on them can, however, be appropriate for simulating the main mode of failure of RC units.
(e) It is pointed out that although the FE model was capable of predicting the emergence of flexural cracks initiating at the bottom and propagating upwards, it was not suitable for predicting their length within the macro-scale mechanics domain. It could, however, be possible to estimate the width of the cracks, based on the elastic and plastic strain results obtained. Crack prediction and growth should be accompanied by good-quality graphics as the small circles used to represent cracks are disappointing and rather misleading.
(f) The experimental model was successfully simulated in the computer using nonlinear FEA and modelling techniques. In general, it has also been demonstrated that current methods and procedures of simulating, assessing, analysing and hence designing in RC can be improved. Further tests are required, combined with more rigorous analytical and numerical work and the establishment of benchmarks, in order to significantly reduce the uncertainties surrounding its performance during their working life.

The FE method is well suited to dealing with composite material models. Consequently, a constitutive RC model based on the theory of plasticity was developed, tested and discussed. One advantage of the theory of plasticity is the relatively simple and direct calibration of the state of stress. The latter results in the yield surface corresponding to a certain stage of hardening, having a strong physical meaning in relation to the strength envelop of concrete. Plasticity theory depends greatly on the existence of a yield surface. This statement is problematic when applied to concrete, as there has been a paucity of associated experimental data until now.

Finally, the choice of a well-established constitutive model in engineering research and practice is important as it affects accuracy. More experimental results and numerical models dealing with complex stress states are necessary for research and general engineering applications in the future.

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Grandstand Terraces.
Experimental and Computational Modal Analysis.

John N Karadelis

8th World Congress on Computational Mechanics WCCM8

5th European Congress on Computational Methods in Applied Sciences and Engineering ECCOMAS 2008

Venice, Italy 30 June – 4 July 2008

INTRODUCTION

- Structural vibrations caused by human activities are not known to be particularly damaging or catastrophic.
- They may be the cause of **concern, discomfort and distress** to occupants or users of the structure.
- Researchers agree: it's a **SLS** problem.
- Research at Coventry University, aims to provide a rigorous interpretation of the behaviour of these structures under dynamic actions, by means of numerical modelling.

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Main Objectives

- Use FE method to accurately **simulate** a group of grandstand terraces.
- **Compare** these models with carefully conducted lab. and/or full scale tests. Highlight inherent uncertainties in simulation and (why not?) experimentation.
- **Alert** on the effect of steel reinforcement on the dynamic properties of these structures and identify any likely patterns related to the above.
- **Investigate and quantify** the effect of support conditions on these properties.
- Use these models for **further**, numerical analysis work.

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BACKGROUND

Modelling real behaviour of RC structures, is not straight forward because of,

- the inherent **non-linear** behaviour of the material,
- the fact that there is no, so far, globally accepted **constitutive law**.

Nevertheless, a respectful number of models have been developed in the past.

- Comprehensive survey of computational „anthology' by Hofstetter & Mang (1995)

Several researchers have made successful (and not so successful) attempts in the past.

- In 1998, Reynolds *et al*, developed a FE model of a post tensioned concrete floor to match their modal test results. However, they did not report on any specific findings resulting from the comparison of their final FE and experimental work.
- Same authors, a year later (1999), attempted the stepped modelling of a multi-storey car park including columns, as opposed to simple pin supports, and claimed better correlation. Again, no limitations or drawbacks were reported. Connections/interfaces were not mentioned anywhere.
- Slender steel columns and their connections can be very "tuneful" and highly "*emotional*". Hence, require accurate computer representation.
- In 2006, Michel and Cunha independently, experimented with ambient type (output only) vibration, obtaining acceptable results for the first two modes.
- The drawback with the ambient vibration is that, methods such as the Fourier based spectral analysis is not appropriate to analyze ambient data because of low SNR (signal-to-noise ratio).
- Ambient, as opposed to input (hammer, shaker, or earthquake) records, have the advantages of being of „infinitely' long duration, „stationary', and in most cases linear. However, they contain an excessively high level of noise making their utilization problematic, or even prohibitive.

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Theoretical Basis

- General equation of motion:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\}$$

- Assuming free vibrations and ignore damping:

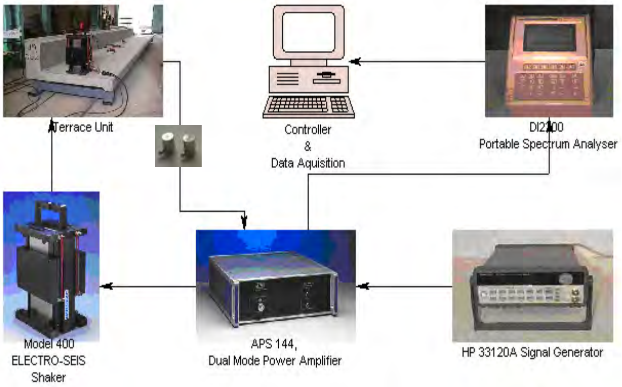
$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\}$$

- Assuming harmonic motion (i.e)

$$\left([K]-\omega^2[M]\right)\{u\}=\{0\}$$

- The roots of this equation are ω_i^2, the eigenvalues, where *i* ranges from 1 to the number of DOF. Corresponding vectors are $\{u\}_i$, the eigenvectors.
- Eigenvalues represent natural frequencies, eigenvectors represent mode shapes (shape assumed by structure when vibrating with its natural frequency.

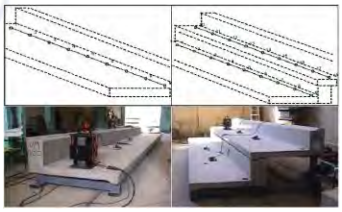
Data Acquisition System



Test Programme & Methodology

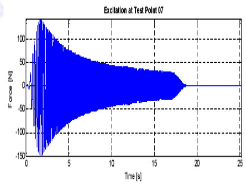
Objective: Was to take measured data, which relate to the *response properties* (eg.: frequency) of a structure and from these to extract the *modal properties* (natural frequencies, etc.)

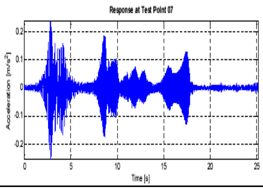
- **Method:**
- Divide each unit into a set of lumped masses "connected" to a set of *'spring/damper'* elements.
- Masses were used as data collection test points (TPs).
- Properties of structure were determined by measuring the FRF between each of the TPs.



- **Procedure:** By shaking, say, TP_1 and measure acceleration at all other mass positions in the structure, the $FRF_{1,1}$, $FRF_{1,2}$, …,$FRF_{1,n}$ were obtained. Natural Freq: determined from the peaks of (FRF) v ω, or ϕ v ω.
- Mode Shapes: by assuming displacement @ TP_1 = 1, and then use displ't transfer functions to determine how all the other TPs move, relative to TP_1.

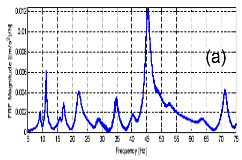
- A *chirp* excitation (short bursts of sine sweeps) was selected for the FRF measurements. Chirp signals allow for a series of averages to be taken with a spectrum analyser over its analysis range. Hence, more representative.
- Chirp excitation swipes from 1 Hz to 70 Hz, while the unit passes through a number of resonance frequencies.
- *Coherence* was used to validate testing (cause and effect signals were collected and compared; accepted when ratio: cause/effect ≈1).
- As motion may be described in terms of displacement, velocity, or acceleration, FRF is called **compliance, mobility, accelerance.**

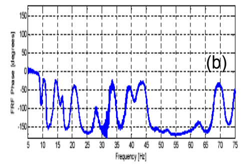




- The term „mobility measurement' is used to describe any form of FRF.

Typical *point mobility* FRF, at TP7 after four frequency domain averages.
(a) FRF peaks and
(b) Characteristic phase changes at frequencies corresponding to the natural frequencies of certain, estimated modes of vibration.





FRF data were imported into *ICATS* for modal parameter estimation and for creating eigen files. The program allows the user to perform „what-if' analysis introducing new masses, bracing, other forms of damping etc.

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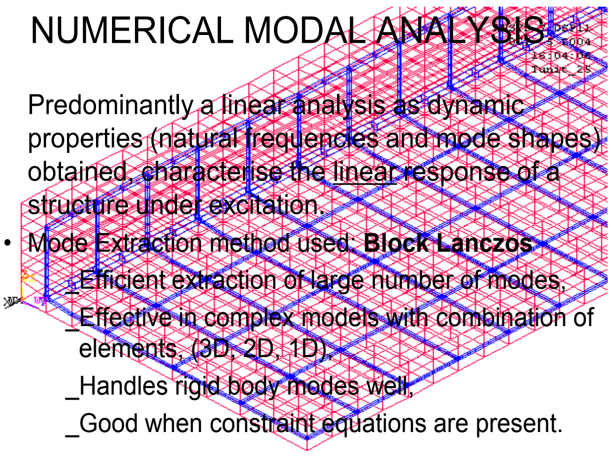
NUMERICAL MODAL ANALYSIS



Predominantly a linear analysis as dynamic properties (natural frequencies and mode shapes) obtained, characterise the linear response of a structure under excitation.

- Mode Extraction method used. **Block Lanczos**
  - _Efficient extraction of large number of modes,
  - _Effective in complex models with combination of elements, (3D, 2D, 1D).
  - _Handles rigid body modes well,
  - _Good when constraint equations are present.

The steel reinforcement

- Inserted in stages, to study its effect on natural frequencies of the unit, as there are conflicting „signals' from a number of researchers!
  Similar studies at MSc level revealed no specific pattern. Demonstrated a tendency to change the dynamic behaviour of a SS-RC beam of rectangular section.
- Noticed (<u>with caution</u>) that considerable increase in the amount of reinforcement (no quantification attempted), is likely to increase certain modal frequencies and decrease others.
  Ie: introducing flexural reinforcement resulted in increasing the first two natural frequencies associated with bending modes. However, had no effect on the next two modes associated with predominantly „torsional' vibrations.
- It seems that, reinforcing and somehow increasing the „specific' stiffness of a particular structural element, may result in "forcing" this element into a different mode of vibration, resulting in lower corresponding natural frequencies.

Modelling Support Conditions

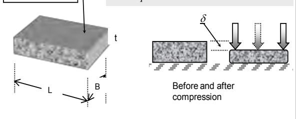
Objective: Seek better correlation between measured and predicted results. Achieved by accurate representation of support conditions.

MODE NUMBER	Measured (Hz)	Damping Ratio (%)	Predicted Plain (Hz)	Predicted Reinforced (Hz)
1	12.000	1.4	17.276	17.708
2	14.700	2.0	29.375	29.380
3	30.000	1.2	41.672	41.373
4	40.000	1.0	76.801	76.933
5	67.300	1.6	122.740	125.010
6			143.170	142.670
7			172.980	171.620
8			186.670	184.270

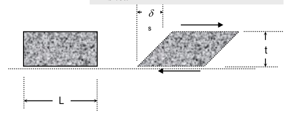
Precast concrete terrace units rest on elastomeric bearings (neoprene pads). They can act as natural vibration absorbers, influencing the vibration performance of the units.

A lab. investigation was launched.

$$K_{comp} = 56.52 \times 10^{6}\ Nm^{-1}$$



$$K_{shear} = 1.248 \times 10^{6}\ Nm^{-1}$$



RESULTS

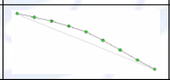
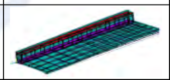
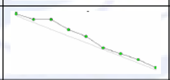
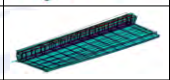
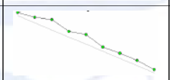
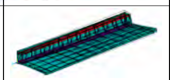
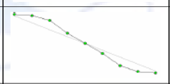
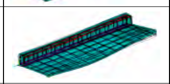
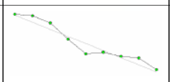
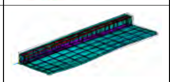
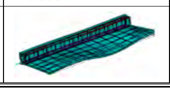
ANSYS provides the user with a variety of elements with stiffness capabilities of which two were set apart:

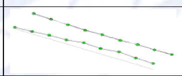
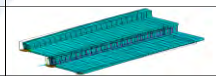
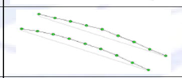
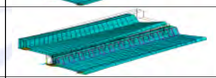
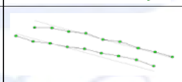
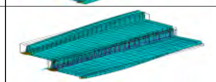
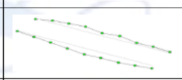
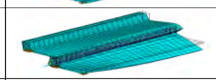
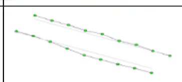
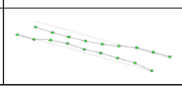
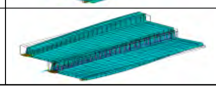
COMBIN14, element with a combination of spring and damper capabilities.

MATRIX27, element whose elastic kinematic response can be specified by stiffness, damping, or mass coefficients. Unique capability to relate two nodes (on structure and on some fixed medium) each with 6DOF, translations and rotations in and about X,Y,Z. Matrices generated by this element are 12x12. After several trials, MATRIX27 was adopted.

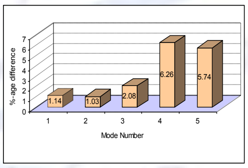
$$\begin{bmatrix} 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & 56.52 & 0 & 0 & 0 & 0 & 0 & -56.52 & 0 & 0 & 0 & 0 \\ & & 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 56.52 & 0 & 0 & 0 & 0 \\ & \text{symmetric} & & & & & & & 1.25 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & 0 & 0 \\ & & & & & & & & & & & 0 \end{bmatrix} \times 10^{6}$$

Mode Number	Measured Damping Ratio (%)	Measured Freq. (Hz)	Predicted Freq. PC (Hz)	Predicted Freq. RC (Hz)	Predicted Freq. RC+NeoP (Hz)
1	1.4	12.000	17.276	17.708	12.243
2	2.0	14.700	29.375	29.380	14.549
3	1.2	30.000	41.672	41.373	30.872
4	1.0	40.000	76.801	76.933	44.560
5	1.6	67.300	122.740	125.010	71.473
6			143.170	142.670	
7			172.980	171.620	
8			186.670	184.270	

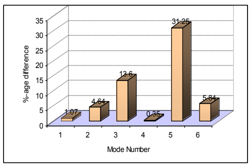
Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	f (Hz) Ex p. Theo . ζ_{exp} (%)	Comments
1			12. 10 12. 24 $\bar{1.4}$	The fundamental bending mode of vibration.
2			14.7 0 14.55 $\bar{2.0}$	Predominantly torsional. May also be showing small amount of bending.
3			30.0 6 30 .70 $\bar{1.2}$	Similar to Mode 2.
4			40. 01 44.56 $\bar{1.0}$	The second (flexural) mode of vibration.
5			67.3 71.4 $\bar{1.6}$	Near perfect flexural mode with a small amount of torsion.
6	?		103.00	Bending Mode

Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	f (Hz) Exp. Theo. ζ_{exp} (%)	Comments (Y, X, Z)
1			9.30 9.40 $\bar{2.2}$	Upper unit is static. Lower unit lifts corner over support (Rigid Body)
2			11.20 11.72 $\bar{1.5}$	First bending & slightly twisting mode. "In phase" vibration
3			16.00 14.08 $\bar{2.5}$	Coupled. Bending & twisting in unison about Z-axis. "In-phase" motion.
4			17.00 17.06 $\bar{2.0}$	Mainly twisting mode about Z-axis "Out-of-phase" Vibration.
5			22.4 29.40 $\bar{2.6}$	Upper unit: "Rigid Body" twisting about Y-axis. Lower unit: Mainly bending.
6			29.1 30.80 $\bar{3.2}$	A complex, mainly torsional, and flexural vibration.

The following charts display the percentage difference between recorded and predicted frequencies per mode number for:



Single terrace unit



Double terrace unit

- No specific pattern emerged at this stage.
- Best agreement (at least for the first three modes of vibration) is achieved for the single unit, as expected.
- Certain findings from the single and double terrace units could be fed to the next stage: Modal Analysis of a (*mock*) Grandstand.

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Conclusions

- Overall, experimentally obtained natural frequencies were in good **agreement** with those predicted by the FE model.
- Experimental modal analysis alone may not be adequate, from the reliability point of view, unless a great deal of **experience** in interpreting results, combined with state-of-the-art **equipment** is available.
- FE analysis should serve as an **aid** and should supplement the former in an effort to explain more accurately the resulting mode shapes especially at higher modes of vibration. In fact, experimental and FE analyses should **interact** with each other for more accurate and successful results.
- Initial and short term tests and results suggest that the amount of reinforcement has only **very little** effect on the dynamic properties of the <u>uncracked</u> reinforced concrete terrace units.
- However, interim studies hinted towards the possibility that an increase in the amount of reinforcement is likely to force the structure into a different mode of vibration, hence **altering** the previously obtained dynamic properties.

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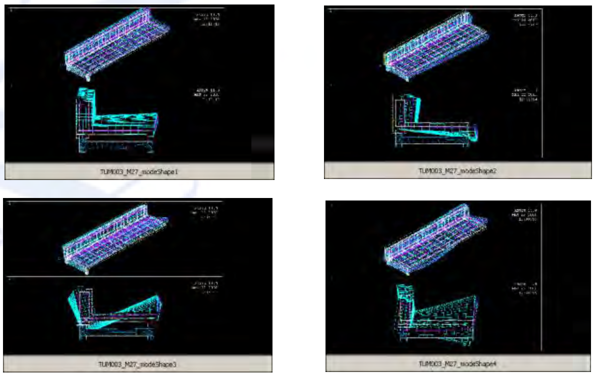
- The dynamic properties of the terrace units were found to be very sensitive to **support conditions** (and/or other connection points, if any), Essentially, it was found that a gradual improvement of the predicted natural frequencies was evident by progressively improving simulation of the boundary conditions.

- Best correlation was achieved when the **behaviour** of neoprene pads was modelled by representing their stiffness characteristics with a matrix, MATRIX27.

- However, there seems to be a **limit**, above which further accuracy is actually **negligible,** in which case one should turn to the experimental methods/procedures to improve accuracy!

- The use of the appropriate number of **triaxial** accelerometers backed with the state-of-the-art **data logging** and **processing** equipment is recommended. However, such equipment is still very expensive and therefore not readily available.

Single Terrace unit.
The first 4 Modes of Vibration



Double Terrace Unit
The First 4 Modes of Vibration

Reliability Pointers for Modal Parameter Identification of Precast Concrete Terraces.

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UK,

ABSTRACT

The reinforced concrete terrace units were positioned and tested on a specially manufactured steel frame resting on the strong floor in the Civil Engineering laboratories at Coventry University.
In parallel, a finite element model was developed and set to free vibration. Natural frequencies and mode shapes were recorded and compared with those obtained experimentally. As correlation was not deemed to be satisfactory, an updating process was initiated and a series of parameters, starting with the concrete material properties were revised to improve links with the experimental results.
Boundary conditions built-in the code were not adequate to model the real behaviour of the structure. Best results were achieved when supports conditions were modelled with a stiffness matrix. Correlation between experimental and computer predicted results improved further with the introduction of more advanced modelling techniques and gradual lifting of the limitations of the model, hence assisting the validation process, while verification did not provide the expected degree of confidence for the model.
It was concluded that it is possible to extract the natural frequencies and mode shapes of a complex, non-symmetric structure accurately, by using relatively low-cost, basic modal testing equipment and the finite element method of analysis, hence avoiding the risk of not detecting any mode shapes. This can be more apparent in complex modes (e.g. coupled, flexural/torsional) as they depend greatly on the number, position, direction, type and quality of the transducers and data logging and processing equipment used.
Emphasis is placed on the experience built up in interpreting modal analysis results in order to be used for future work.

Keywords: Modal analysis, Concrete, Updating, Correlation, Natural frequencies, Mode shapes.

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1. INTRODUCTION

Civil engineering structures (e.g. grandstands), their boundary conditions and interaction with neighbouring structures and sometimes their non-structural elements, constitute complex assemblies playing an important role in their dynamic performance and are difficult to model globally. Hence, testing and modelling smaller units and parts (sub-structuring) can be an acceptable approach towards the goal for more efficient modelling, retrofitting and designing of such structures. This paper argues that the finite element method can take the leading role in the extraction of modal parameters of such structures and can help reduce costly, time consuming and complex full-scale modal tests. The confined objectives can be summarised as follows:

- To improve modal analysis standards in Civil Engineering by proposing a contemporary system identification method, that effectively captures the natural frequencies and mode shapes of grandstand terraces.
- To update, calibrate, compare, validate and verify previously developed „static' models by the author using carefully conducted full scale modal tests and an analytical solution and then highlight any inherent uncertainties in simulation analysis and experimentation.
- To investigate and quantify the most effective parameters affecting the dynamic properties of the grandstands.
- To comment on the effect of steel reinforcement in the dynamic properties of these structures and identify any inherent patterns that may be related to the above.

2. MODAL ANALYSIS OF GRANDSTANDS (Brief overview)

The basic assumptions that have to be made to perform an experimental modal analysis of any structure are:

1. The structure is assumed to be linear, obeying the principle of superposition.
2. The structure is time invariant, meaning that the parameters to be determined are constant and do not change with time.
3. The structure obeys Maxwell's Reciprocity Theorem.

Experimental modal identification techniques can be divided into two main categories: Input-Output and Output only and they are almost always accompanied by finite element correlation and updating, hence ensuring that the final numerical models reflect better the measured data than the initially developed ones.

The hybrid (steel skeleton-concrete terraces) method of stadia construction can be susceptible to excessive vibration caused by their users (Bachmann, 1983), Allen 1990, Batista and Magluta 1993). These vibrations are judged excessive by the discomfort, or even the panic and alarm they can cause to the users (serviceability limit state problem) or in extreme cases, causing low cycle fatigue failure (ultimate limit state problem).

The Joint Working Group (JWG) of the Institution of Structural Engineers in UK was established in January 2000 to consider, advise and make recommendations on the dynamic performance and design of stadia structures (Interim Guidance 2001, Advisory Note 2002, Note 2003). The Group completed their work by identifying a number of areas where more research should be directed. One of these areas was the need for accurate numerical modelling of the structures in question.

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More pioneering work has been dedicated to the subject such as a series of publications by Ellis and Littler (2004a, 2004b). Earlier, Ellis and Ji (1996) were involved in the estimation of the dynamic properties of structures. They concluded that the experimental study provided quite accurate but incomplete information while the theoretical study supplied complete but inaccurate results and stressed for more accurate computer simulation.

Reynolds et al (2004a, 2004b) developed a remote monitoring system to measure the vibration performance of stadia when empty and in-service during sporting and other events. They too, pointed towards the absence of accurate finite element modelling procedures. The same researchers, (Reynolds *et al* 2005) investigated the dynamic behaviour of the City of Manchester Football Stadium stressing the need for more accurate, 3D FE modelling to capture the real behaviour of the whole stand.

Swan, *et al* (2005) established that support conditions are effective in influencing the Frequency Response Function (FRF) measured on a model deck and that the FRF may be used as an accurate general indication of the overall state of stiffness of a bridge's deck. However, their methodology incorporated elementary support modelling techniques only. Their subsequent consideration of a series of springs created unwanted alignment problems that hindered the process and affected the results. Hence, they were not able to report on the degree of effectiveness and expand on the structure's stiffness.

Finally, Ibrahim and Reynolds (2007) demonstrated good correlation of natural frequencies and mode shapes and suggested that progressive modelling of different configurations is a fairly accurate approach in the FE modelling of large structures such as grandstands.

It is apparent from the above that accurate and reliable finite element models to complement and verify the experimental identification and estimation of the modal parameters are in short supply today. An up to date numerical model and a contemporary modal parameter identification method will be reported in this study emphasizing that some civil engineering structures (grandstands), their boundary conditions and non-structural elements are too large and complex to be modelled at a meso/macro-scale level. Therefore sub-structuring, modelling (and testing) smaller units, can show the way to better computer representation of such structures.

3. THEORETICAL BASIS

In general, it is assumed that the structure has constant stiffness and mass effects and that there is no damping present. No constant or time dependent forces or displacements are applied (free vibration). Hence, the eigenvectors are the displacement solutions of the equilibrium equation of motion for free, undamped vibrations:

$$[M]\{\ddot{x}\}+[K]\{x\}=0 \tag{1}$$

where: $[K]$ = stiffness matrix
$[M]$ = mass matrix
$\{x\}$ = displacement vector

For a „linear' structure the displacement is harmonic, of the form:

$$\{x\} = \{\phi\}\sin\omega t \tag{2}$$

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where: $\{\phi\}$ = vector of order n (amplitude)
ω = natural circular frequency of vibration

Equation (3) shows the classic eigenvalue problem used in a typical undamped modal analysis.

$$[K]\{\Phi_i\} = \omega_i^2[M]\{\Phi_i\} \tag{3}$$

where: $\{\Phi_i\}$= mode (shape) vector (eigenvector) of mode i
ω_i = natural circular frequency of mode i (ω_i^2 is the eigenvalue)

It can be shown easily that:

$$[\Phi]^T[M][\Phi] = [1] \tag{4}$$

$$[\Phi]^T[K][\Phi] = [\Omega^2] \tag{5}$$

Equations (4) and (5) demonstrate the two „special' properties of the eigenvectors. Each eigenvector is associated with a particular eigenvalue, both being special properties of square matrices. This constitutes the basis of any numerical modal analysis code.

In the case of the terrace units having no plane of symmetry, the problem becomes significantly more complex and involves coupled torsional and flexural vibrations in the two principal planes. Weaver *et al* (1990) warned of the development of three simultaneous differential equations instead of two and radically more elaborate analysis and computations. The entire, detailed analytical model is presented elsewhere (Karadelis 2010) but an insight is outlined below, based on Figure 1.

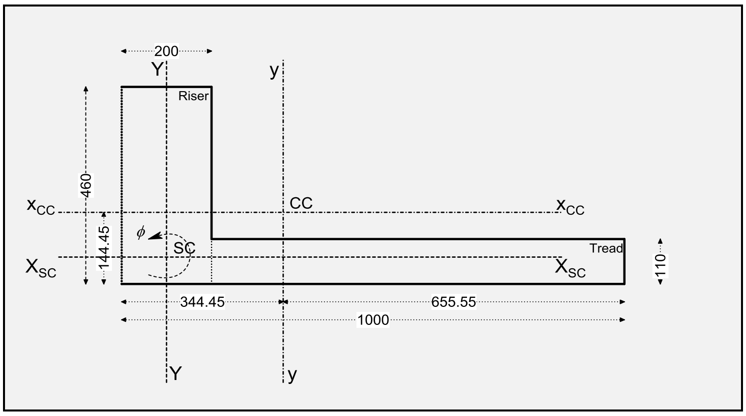

Figure 1. Cross-section of a terrace unit. CC = centroid, SC = shear centre.

The differential equation of flexure when bending is constrained in the vertical and horizontal planes respectively is given in statics by:

$$EI_x \frac{d^4v}{dz^4} = w_y \tag{6}$$

$$EI_y \frac{d^4u}{dz^4} = w_x \tag{7}$$

where:

EI_x, EI_y= flexural rigidity about a horizontal and vertical axes respectively.
v, u= displacements in the vertical and horizontal directions
w_y, w_x= intensity of distributed load in Y and X directions
z= longitudinal (along the main span) direction

Assuming torsion takes place about the shear centre, SC (Timoshenko 1955),

$$T_{(SC)} = GJ\frac{d\varphi}{dz} - EI_w\frac{d^3\varphi}{dz^3} \tag{8}$$

Or, after differentiation:

$$we_x = GJ\frac{d^2\varphi}{dz^2} - EI_w\frac{d^4\varphi}{dz^4} \tag{9}$$

where:

GJ= torsional rigidity
EI_w= warping rigidity
φ = angle of twist
we_x= intensity of torque

The equations of motion (EoM) can be put together such as:

$$\left.\begin{aligned}
&EI_y\frac{\partial^4u}{\partial z^4} + EI_{xy}\frac{\partial^4v}{\partial z^4} + \rho A\frac{\partial^2u}{\partial t^2} - \rho Ae_y\frac{\partial^2\varphi}{\partial t^2} = 0\\
&EI_y\frac{\partial^4v}{\partial z^4} + EI_{xy}\frac{\partial^4u}{\partial z^4} + \rho A\frac{\partial^2v}{\partial t^2} + \rho Ae_x\frac{\partial^2\varphi}{\partial t^2} = 0\\
&EI_w\frac{\partial^4\varphi}{\partial z^4} - GJ\frac{\partial^2\varphi}{\partial z^2} + \rho Ae_x\frac{\partial^2v}{\partial t^2} - \rho Ae_y\frac{\partial^2u}{\partial t^2} + \rho I_{sc}\frac{\partial^2\varphi}{\partial t^2} = 0
\end{aligned}\right\} \tag{10}$$

where:

u, v = displacements of the shear centre, SC, in X, Y directions
ρ = mass density
A = cross-sectional area
e_y, e_x = distances from the centroid, CC to X and Y (shear centre) axes respectively.
I_{sc} = polar moment of inertia about the SC
EI_{xy} = coupling stiffness (rigidity)
t = time

The following terms,

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$\left[\rho A \frac{\partial^2 u}{\partial t^2}\right]$, $\left[\rho A \frac{\partial^2 v}{\partial t^2}\right]$, $\left[\rho A e_y \frac{\partial^2 \varphi}{\partial t^2}\right]$, $\left[\rho A e_x \frac{\partial^2 \varphi}{\partial t^2}\right]$ denote mass and inertia associated with linear and angular acceleration respectively,

$\left[E I_{xy} \frac{d^4 u}{dz^4}\right]$, $\left[E I_{xy} \frac{d^4 v}{dz^4}\right]$ indicate coupling effects,

$\left[E I_w \frac{\partial^4 \varphi}{\partial z^4}\right]$, $\left[G J \frac{\partial^2 \varphi}{\partial z^2}\right]$ are the warping and torsional terms respectively,

$\left[\rho A e_x \frac{\partial^2 v}{\partial t^2}\right]$, $\left[\rho A e_y \frac{\partial^2 u}{\partial t^2}\right]$ represent inertia forces and

$\left[\rho I_{sc} \frac{\partial^2 \varphi}{\partial t^2}\right]$ represents inertial torque.

After processing the EoM, the boundary conditions are introduced (z=0 and z= ℓ) and the Frequency Equation (FEq) can be structured by setting the determinant of Eqs. (10) equal to zero ($\Delta_{(\mathrm{Eqn.10})}$= 0). Following a vast amount of mathematical computations not shown here, the final FEq with respect to ω yields:

$$\boxed{\begin{aligned}
&\omega^6\rho^3A^2\left\{A\left(e_x^2+e_y^2\right)-I_{sc}\right\}+\\
&+\omega^4\rho^2\left\{AEI_{sc}\frac{k^4\pi^4}{\ell^4}\left(I_y+I_x\right)-A^2E\frac{k^4\pi^4}{\ell^4}\left(I_ye_x^2+I_xe_y^2\right)+A^2EI_w\frac{k^4\pi^4}{\ell^4}-\right.\\
&\qquad\left.-2A^2EI_{xy}e_xe_y\frac{k^4\pi^4}{\ell^4}+A^2GJ\frac{k^2\pi^2}{\ell^2}\right\}-\\
&-\omega^2\rho\left\{E^2I_{sc}\frac{k^8\pi^8}{\ell^8}\left(I_xI_y-I_{xy}^2\right)+AE^2I_w\frac{k^8\pi^8}{\ell^8}\left(I_y+I_x\right)+AEGJ\frac{k^6\pi^6}{\ell^6}\left(I_y+I_x\right)\right\}+\\
&+E^3I_w\frac{k^{12}\pi^{12}}{\ell^{12}}\left(I_xI_y-I_{xy}^2\right)+E^2GJ\frac{k^{10}\pi^{10}}{\ell^{10}}\left(I_xI_y-I_{xy}^2\right)=0
\end{aligned}} \tag{11}$$

Finally, as the various constants are evaluated and substituted in Eq. (11), the latter yields the following results for k= 1,2,…6.

Table 1 Calculated values of (first six) natural frequencies.

Calculated ω rad.s^{-1})	Calculated f (Hz)
75.23	11.98
93.57	14.90
199.70	31.80
275.37	43.85
444.00	70.70
704.86	112.24

4. FULL SCALE MODAL TESTING

Tests were carried out partly in the Civil Engineering laboratories of Coventry University and partly at Bison's headquarters in Slough, England. The scope of this paper is not to present a detailed account of the experimental programme. This is given elsewhere (Karadelis 2010). However, some essential material is reported here.

4.1 The Vibration Test System

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The essential components of the modal identification system used are shown in Figure 2. In this case the structure under tests is a terrace unit. The Shaker is used to deliver the input force. The Amplifier provides power to the shaker's armature coil. The Signal Generator generates and drives the signal (sine, square, ramp, even earthquake simulations) to the shaker. The Spectrum (FFT) Analyser, performs time and frequency domain analysis, calculates the frequency response function (FRF) and exports all data to a personal computer. Finally, the Accelerometers (not shown here) measure the structure's response.

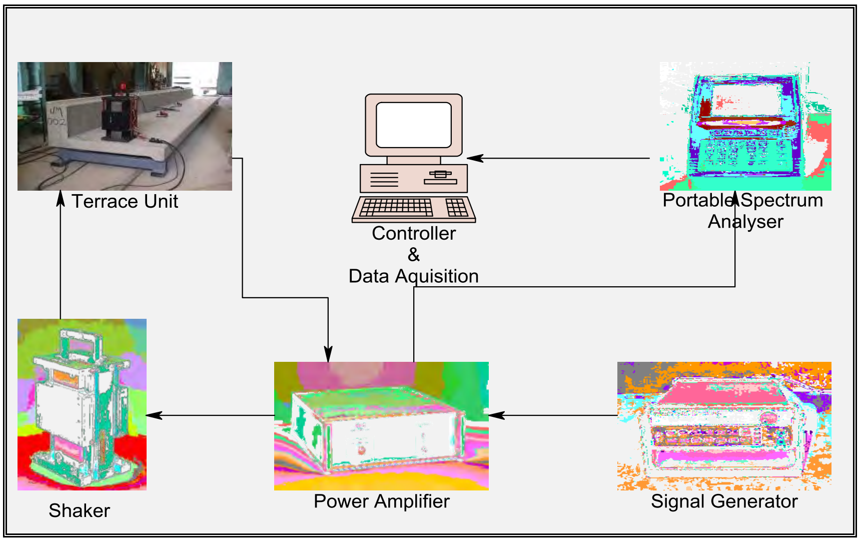

Figure 2: Modal Analysis Data Acquisition Test System.

4.2 Test Programme and Methodology

The objective was to take measured data, which relate to the response properties (such as the input frequency) of a structure and from these to extract the modal properties (natural frequencies, etc). The terrace unit was modelled by dividing it into a set of lumped masses shown in Figure 3. These masses are intended to be connected to a set of spring/damper elements. In modal analysis, masses are used as data collection (reference) points (RPs) and the properties of the structure are determined by measuring the FRF (Frequency Response Function) at each of the RPs.

The FRF was measured by shaking, say, RP1 and measuring acceleration at other mass positions in the structure. This would yield $FRF_{1,1}$, $FRF_{1,2}$, $FRF_{1,3}$…, $FRF_{1,n}$. Given the frequency response functions, it is possible to determine the natural frequencies and damping by locating the peaks of the FRF v Frequency plot or the Phase Shift v Frequency. The mode shapes are then determined by assuming a unit deflection at, say, RP1 and use displacement transfer functions to determine how all the other points will move relative to point 1. The response of the unit was measured at two locations, RP3 and RP5. To ensure linearity, a sample of the amplitudes reached during the tests were compared with the allowable static deflections and were found to be well within range.

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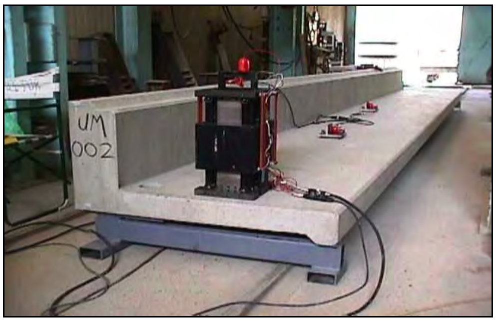


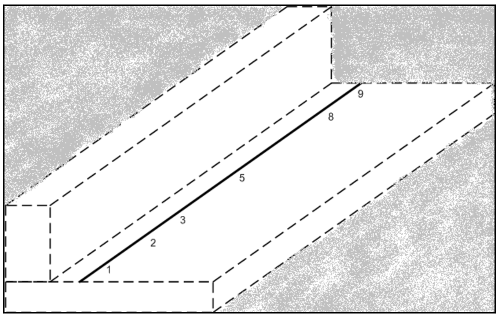
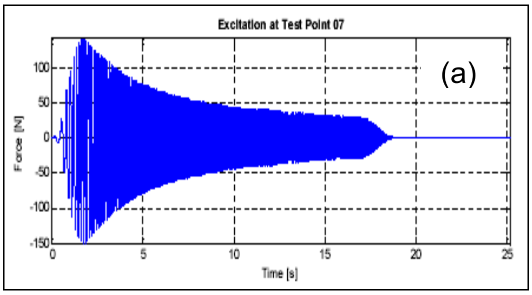


Figure 3 A terrace unit in the laboratory and its corresponding modal test grid.

A chirp excitation was selected for the FRF measurements. Chirp signals are comprised of short bursts of sine sweeps. They allow for more rapid tests than the sine sweeps, as a series of averages can be taken with a spectrum analyser over its analysis range, rather than requiring the sine sweep to reside at each frequency. Coherence was used to validate the testing. That is, by using the multi-channel spectrum analyser both, the cause and the effect signals were collected and compared and only when their ratio was found to be near 1.0, were accepted. A summary of the main data acquisition parameters is given in Table 2 and a typical example of excitation and response time history is shown in Figure 4. There is a clear exaggeration at certain points in the time history provided by the response accelerometer (b). These exaggerations correspond to the unit passing through a number of resonance frequencies as the chirp excitation (a) swipes from 1 Hz to 70 Hz.

Table 2 Data acquisition parameters for normal test FRF measurements.

PARAMETER	Setting/Value
Acquisition Bandwidth (Sampling Rate):	80 Hz (325.5 Hz)
Acquisition Duration:	25.00 s
Frequency resolution:	0.0397 Hz
No. of frequency Domain Averages:	4
Excitation Type:	Chirp
Excitation Duration:	18.87 s
Excitation Frequency Span:	1 –79 Hz



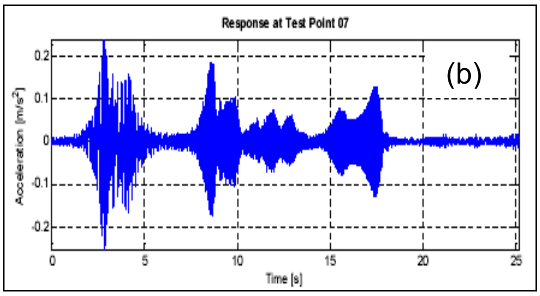


Figure 4 Typical excitation (a) and response (b) signals on the single unit.

In general, motion can be described in terms of displacement, velocity or acceleration. In this case the corresponding FRF is called compliance, mobility or accelerance. However, the term „mobility measurement' is used to describe any form of FRF. Figure 5 below shows a typical point mobility FRF, after four frequency domain averages, estimated experimentally in the vertical direction, at RP7. The FRF peaks (a) and the characteristic phase changes (b) at frequencies corresponding to the natural frequencies of certain estimated modes of vibration are visible.

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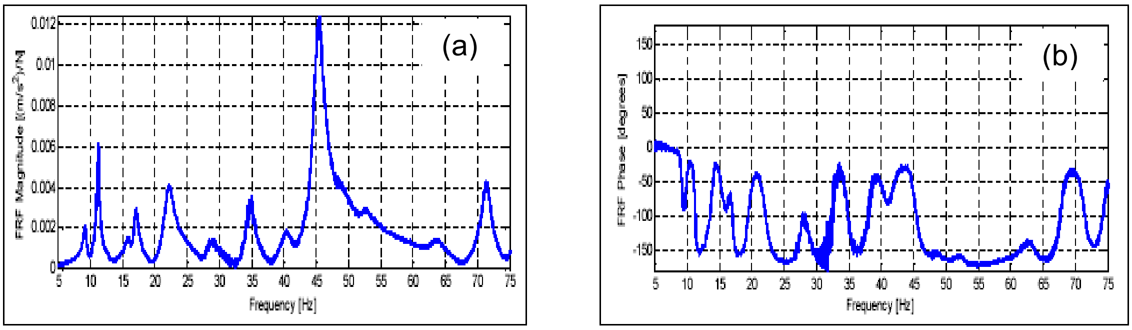

Figure 5 Typical FRF at RP7, after four averages.

Modal parameter estimation was performed using ICATS (1997) after importing all FRF data into it.

5. NUMERICAL MODAL ANALYSIS

Modal Analysis is a linear analysis as the dynamic properties (natural frequencies and mode shapes) obtained, characterise the linear response of a structure under excitation. The initial, static numerical model used in this study has been published elsewhere (Karadelis 2009a, 2009b). However, essential matter such as a brief account of the model built-up, material and geometric properties, boundary conditions, etc, particularly in conjunction with the process of correlation, will be reported here to aid the reader. Also, the method of extracting the mode shapes is of interest and will be briefly addressed.

5.1 The Initial, 'Static' Model

The terrace units (Figure 3) were designed, manufactured and transported to the laboratory by Bison Concrete Products Ltd. They were designated *uncracked* upon delivery. Table 3 summarises the basic properties.

Table 3. Initial (design stage) material properties of the terrace units.

Material Properties			
Characteristic Concrete Strength, f_{cu}= 45 Nmm^{-2} (C35/45)			
Reinforcement (Flexural, T&C) Characteristic Strength, f_y= 460 Nmm^{-2}			
Reinforcement (shear) Characteristic Strength f_{yv}= 460 Nmm^{-2}			
Loading			
Dead Load (self-weight) = 3.65 kNm^{-2}			
Imposed Load = 4.00 kNm^{-2}			
Main Reinforcement			
Riser	**Bar size**	**Tread**	**Bar size**
Longitudinal Reinforcement. Top:	2T12	Long. Span:	T16 @ 150 c/s
Longitudinal Reinforcement. Bottom:	2T25	Short. Span:	T8 @ 150 c/s
Shear Links:	T6 @ 150 c/s		

The Modulus of Elasticity of concrete was taken from relationships and tests shown condensed in Table 4. The Modulus of Elasticity of high yield steel reinforcement and its 0.2% proof stress were estimated by routine laboratory tests (Karadelis 2009a).

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Table 4. Measured and calculated Modulus of Elasticity of Concrete and Steel in kN/mm^2

E_{Hughes}	E_{static}	$E_{u/sonic}$	E_{BS8110}	E_{ave}
$9100\,(f_{cu})^{1/3}$ 32.364	cylinder compr. tests 30.04	ultrasonic lab. tests 29.80	value given by BS8110 33.5	average value 31.50
E_{steel} estimated in lab. 198.96	E_{steel} value for design 200.00	0.2% proof stress (estimated in lab.) 525.00		

The initial model was revived from earlier, similar studies in the statics domain using ANSYS (2005) code. Its purpose was to serve as a preliminary model for further FE updating. The model was based on the SOLID65 element chosen to represent concrete. This is a 3D, eight node, solid, isoparametric element with three translational degrees of freedom (DOF) per node. When in a non-linear domain, it is capable of simulating the brittle behaviour of concrete. This property vanishes when the element is used in a linear environment like a modal analysis. A discrete representation was assigned to steel by involving LINK8, 1D elements capable of resisting tension and compression. Shear was modelled in a smeared manner (Karadelis 2009b).

Previous studies and personal experience of the author have suggested that Poisson's ratio is not sensitive enough to cause notable changes in the behaviour of concrete structures either in the static or the dynamic domain (Hughes and Karadelis 1998, Reynolds *et al*, 2002). On the other hand, any variations in the density of the two materials can cause considerable changes in the dynamic behaviour of the units.

Other parameters (in addition to physical properties) may influence the updating process and the quality of results such as the supports (or any boundary) conditions, the steel reinforcement, even the solution procedure used by the program. Table 5 lists the physical properties of steel and concrete as used in the initial non-lineal, static, finite element model.

Table 5. Physical properties of concrete and steel used in the „static' model.
$E_{1t,\,con}$ refers to the initial tangent modulus of concrete.

Parameter	Value
$E_{1t,con}$	30.00 (kN/mm^2)
ρ_{con}	2240 (kg/m^3)
ν_{con}	0.225
E_{steel}	198.960 (kN/mm^2)
ρ_{steel}	7750 (kg/m^3)
ν_{steel}	0.300

5.2 Updating Strategy, Correlation, Fine-tuning, Validation.

The goal of FE model updating is to achieve an improved match between model and test data by making physically meaningful changes to model parameters and therefore rectify any inaccurate modelling assumptions. Updating methods are based primarily on the sensitivity of selected physical parameters so that correlation between simulated responses and target values improves. Figure 6 summarizes the updating strategy in a symbolic, brief flow-chart.

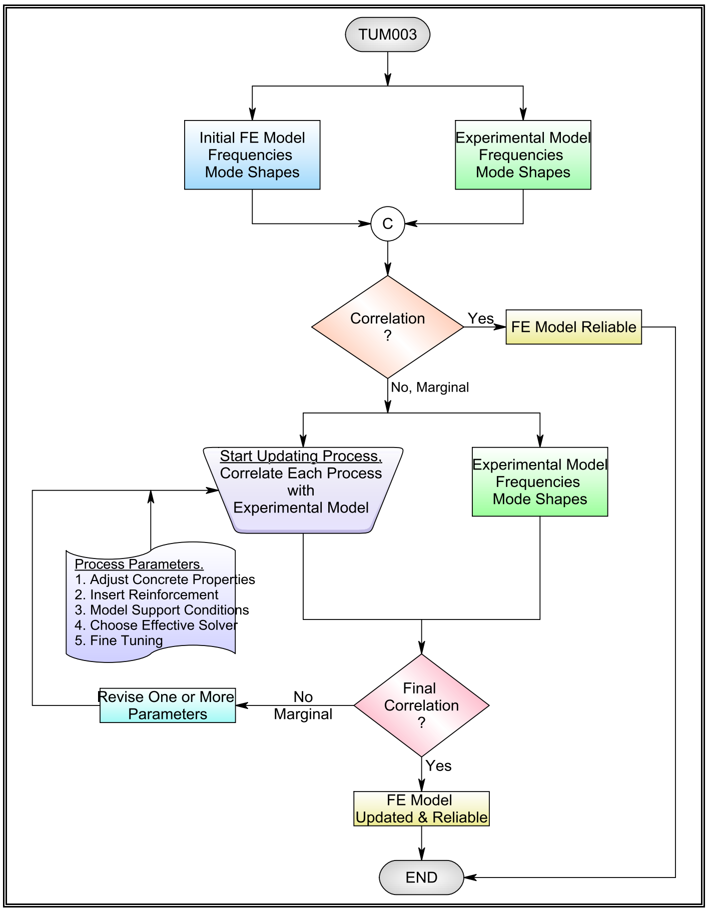

Figure 6 The updating process summarised.

The main difference between computational and manual updating is that the former allows for a simultaneous variation of more than one parameter and therefore should be a good deal faster. A common obscurity in computational model updating is the proper choice of appropriate model parameters and although automated methods may be used they do not always deliver reliable results. The response can be correlation values like MAC (Modal Assurance Criterion), the most popular criterion for correlating vectors in a computer driven FE model updating. The purpose of a successfully updated FE model is to be used for further structural analysis, or model other loadings, boundary conditions, or even different configurations (such as damage) with much more confidence and without additional experimental testing.

Gradual Updating and Correlation

A plain concrete model taken from a previous static analysis of the same terrace units and with the material properties shown in Table 7, below was used as an opening case. The results predicted are shown compared with those collected from tests. The percentage error is also shown but it is based on the experimental (measured) results which can only be assumed to be accurate. The fact that all accelerometers were positioned vertically on the structure and therefore not make up for other directions may also account for that. A more realistic indicator, such as the percentage deviation from the arithmetic mean of the two values is also shown. This may be more erroneous than the percentage error at the start of the updating process but should become more accurate at a later stage when correlation improves. A first attempt to correlate the measured and predicted results is plotted in Figure 7.

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5.2.1 Initial FE Model

Table 7 Preliminary FE model. Measured and predicted natural frequencies.

$E_{concrete}$ = 30.00 (kN/mm^2) $\rho_{concrete}$ = 2250 (kg/m^3)					
Mode No.	**Measured ξ. (%)**	**Measured *f*. (Hz)**	**Initial *f*. (Hz) Plain Concrete**	**% Error**	**% Dev**
1	1.4	12.0	17.28	43.97	18.02
2	2.0	14.7	29.38	99.83	33.30
3	1.2	30.0	41.67	38.91	16.29
4	1.0	40.0	76.80	92.00	31.51
5	1.6	67.3	122.74	82.38	29.17
6			143.17		

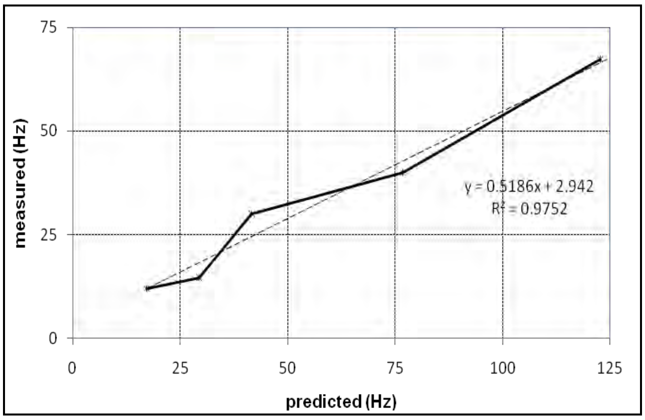

Figure 7. Preliminary correlation between measured and predicted natural frequencies.

As it is expected in practical applications like the above, the composition of the first finite element model will not be correct; hence the updating process. However, sometimes it is possible that the final parameter changes do not allow for a physical interpretation. They are simply numerical substitutes accountable for reducing the deviations between tests and FE analysis. Whether the latter are physically acceptable or not depends on the field of application of the model. For example, a large, disproportionate increase in the thickness of shell elements may be unacceptable for a static stress analysis but has no effect on modal analysis. Hence, in a manually updating process it is left to the analyst to justify and decide whether the resulting model is acceptable and realistic or not. The slope of the „best-fit' was chosen as a correlation index. The initial slope was 0.518. This is out by 0.482, or 48.2%, from perfect correlation (target), taken as one.

5.2.2 First Update. (Material Properties)

It has been said that finite element analysis overestimates stiffness (Hyo-Gyoung Kwak and Filippou 1990, Ha-Wong Song *et al* 2002, Serif and Dilger 2006). This is usually attributed to overestimation of material properties, the variation of stiffness in different directions and interaction problems between two neighbouring materials, as it is shown later. Hence, as stiffness is directly related to the square of the natural frequency, it is expected that FE Analysis will overestimate the natural frequencies. The first update was based on the variation of material parameters. Hence, the inserted value for Young's Modulus of concrete was lowered and the corresponding value for density was raised in an effort to come near the measured natural frequencies. Both variations were performed within the acceptable boundaries of the parameters themselves. Table 8 and Figure 8 display the details and the results.

Table 8. First FE update. Measured and predicted natural frequencies

$E_{concrete}$ = 29.00 (kN/mm^2) $\rho_{concrete}$ = 2400 (kg/m^3)					
Mode No.	**Measured ξ. (%)**	**Measured f. (Hz)**	**1st FE-update f. (Hz) (Mat Props)**	**% Error**	**% Dev**
1	1.4	12	15.32	27.67	12.15
2	2.0	14.7	21.83	48.50	19.52
3	1.2	30	38.75	29.17	12.73
4	1.0	40	64.05	60.13	23.11
5	1.6	67.3	91.25	35.59	15.11
6			112.70		

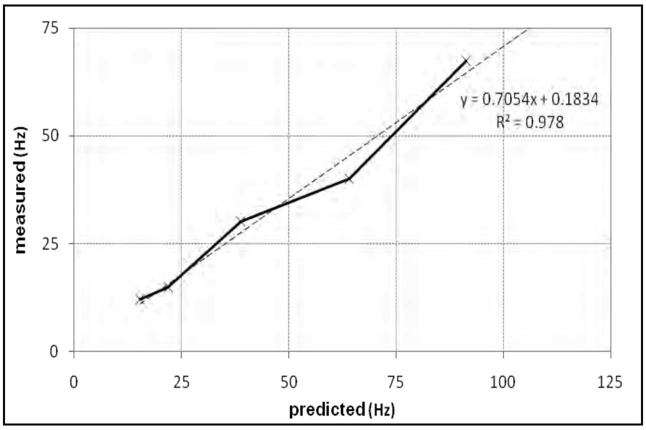

Figure 8. First correlation between measured and predicted natural frequencies.

The results show a noticeable improvement in the predicted variables and a reduction in the percentage error and percentage deviation compared with the initial model. The slope of the „best fit' curve has been improved from 0.518 to 0.705; an upgrading of 0.187 towards the target value, or 0.295 (29.5%) away from unity. The largest deviation occurred at mode 4 and the smallest at mode 1, as expected.

5.2.3 Second Update (Reinforcement).

Physical properties from the first update were kept constant during the second updating process while steel reinforcement was introduced in the model. It was decided to test the effect of steel reinforcement on the natural frequencies of the units as there are currently confusing arguments regarding the above (Numayar *et al* 2003). The reinforcement was introduced gradually in order to observe the response of the unit. Table 3 shows the amount and type of reinforcement used. Note that distribution steel, warping control steel at corners, overlapping steel, as well as some other "features" of reinforced concrete design were not modelled. The second update was carried out and the following results were collected in Table 9 and Figure 9.

Table 9. Second FE update. Measured and predicted natural frequencies.

Mode No.	Measured ξ. (%)	Measured f. (Hz)	2^{nd} FE-update f. (Hz) (Reinforcement)	% Error	% Dev
1	1.4	12	15.44	28.67	12.54
2	2.0	14.7	21.1	43.54	17.88
3	1.2	30	38.6	28.67	12.54
4	1.0	40	64	60.00	23.08
5	1.6	67.3	85.5	27.04	11.91
6			109		

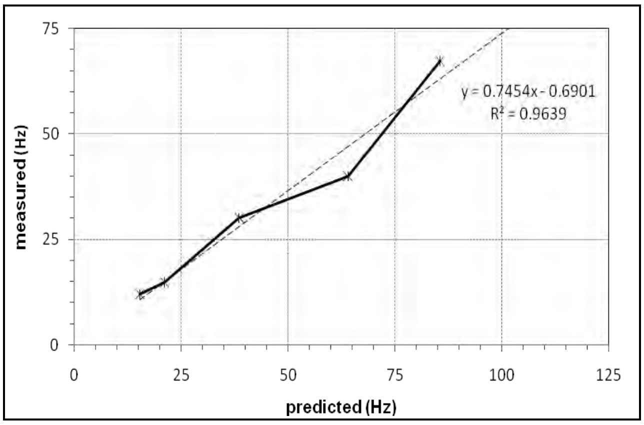

Figure 9. Second correlation between measured and predicted natural frequencies.

Depending on the particular mode of vibration, the percentage error and percentage deviation has both increased and decreased. Modes 1, 2, 4 had their deviation increased but modes 3 and 5 had their own decreased. This shows that the reinforcement does have an effect on the natural frequencies but not clear pattern for useful conclusions has yet been emerged. This is discussed in the next section. The gradient of the best fit has improved from 0.705 to 0.745, a marginal improvement of 0.04. The best-fit improved to 0.255, and it is now 25.5% away from the target.

5.2.4 Third Update. (Support Stiffness)

The third update involved the accurate simulation of supports in the finite element model. Physical properties and steel reinforcement were kept constant this time. Support (boundary) conditions play an important role in the dynamic response of any structure. Initially, all 3 translational DOFs were restrained at one end of the terrace unit simulating a 3-dimentional pin (UX= UY= YZ= 0), using ANSYS' built-in facility. In addition, symmetric conditions at midspan were applied with some caution. That is, a full model with a coarse mesh was used and its first six mode shapes were matched with those obtained from a symmetric model in order to make sure that any odd, non-symmetric modes would not be missed.

The objective was to achieve better correlation between measured and computer predicted results. It is known that precast concrete terrace units usually rest on elastomeric bearings (bearing pads). Briefly, the pad's multi-function is:

- To protect the concrete from spalling;
- to transfer loads smoothly and uniformly;
- to allow for some end rotation over the supports;
- to cater for differential vertical movement and levelling problems;

- to allow and compensate for some lateral and longitudinal „thermal' movement;
- to act as natural vibration absorbers influencing the vibration of the units.

British Standard 2752 (2003) provides some information regarding specifications of chloroprene rubber compounds. ANSYS provides the user with a variety of elements with stiffness capabilities. The MATRIX27 element was chosen due to its unique aptitude to allow the analyst full control of its input parameters, and its ability to relate two nodes (one lying on the structure and the other on some fixed medium) each with 6DOF, translations along and rotations about, the nodal x, y, z axes. All matrices generated by this element are 12x12 matrices. Stiffness values were evaluated using relationships from Lindley (1966). The procedure is rather lengthy but straight-forward and therefore not shown here. The following stiffness values were obtained and checked against the manufacturer's data-sheets. Ed. (12) shows the stiffness matrix developed specifically for ANSYS solver.

Compressive stiffness of the pad, $\mathbf{K}_{\mathbf{comp}} = 56.52 \times 10^6 \,\mathrm{Nm}^{-1}$

Shear stiffness of the pad, $\mathbf{K}_{\mathbf{shear}} = 1.248 \times 10^6 \,\mathrm{Nm}^{-1}$

$$\mathbf{K} = \begin{bmatrix} 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & 56.52 & 0 & 0 & 0 & 0 & 0 & -56.52 & 0 & 0 & 0 & 0 \\ & & 1.25 & 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & -1.25 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 56.52 & 0 & 0 & 0 & 0 \\ & & \text{symmetric} & & & & & & 1.25 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & 0 & 0 \\ & & & & & & & & & & & 0 \end{bmatrix} \mathrm{x}10^6 \qquad (12)$$

The resulting natural frequencies predicted by ANSYS are cited in Table 10 and Figure 10 for comparison with the measured results.

Table 10. Third FE update. Measured and predicted natural frequencies.

Mode No.	Measured ξ. (%)	Measured *f.* (Hz)	3rd FE-update *f.* (Hz) (Support-Stiff)	% Error	% Dev
1	1.4	12	12.74	6.19	3.00
2	2.0	14.7	15.00	2.04	1.01
3	1.2	30	31.72	5.73	2.79
4	1.0	40	44.56	11.40	5.39
5	1.6	67.3	71.47	6.20	3.01
6			100.20		

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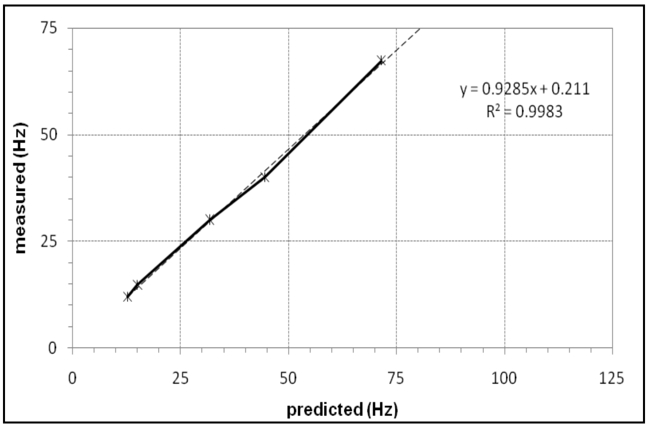

Figure 10. Third correlation between measured and predicted natural frequencies.

It is obvious that a more accurate support representation had a significant effect in correlation. The slope of the best fit has changed from 0.745 to 0.918, an improvement towards target of 0.173 and only 0.082 (8.2%) away from perfect correlation.

5.2.5 Forth Update (Block Lanczos)

ANSYS provides the user with a variety of different solution techniques. The subspace method was initially chosen in the present study. This uses the generalised Jacobi iteration algorithm (Mahinthakumar and Hoole 1990) which is similar to the familiar Gauss-Seidel iteration but consists in not using improved values until a step has been completed.

For large, symmetrical problems ANSYS recommends the Block Lanczos eigenvalue extraction method (Lanczos 1950). This uses the Lanczos algorithm performed with a block of vectors, as opposed to a single vector (Lewis *et al* 1994). This solver performs particularly well when the model consists of a combination of 3D and 2D or 1D elements. It uses the sparse matrix solver, overriding any other solver specified. The Block Lanczos method is especially powerful when searching for eigen-frequencies in a specific part of an eigenvalue spectrum of a system. The subsequent adoption of Block Lanczos method for this study has significantly reduced the CPU-time and has added to the accuracy of the results over the initial choice of the subspace method. Table 11 and Figure 11 show the results from the modal analysis. As the predicted results converge gradually towards the measured ones, the rate of the percentage deviation is reducing. Also, the difference between the last and present best fit is reducing (from 0.918, to 0.936), a modest improvement of just 0.018 and the „distance' from the target is now only 0.064 or 6.4%.

Table 11. Forth FE Update. Measured and predicted natural frequencies.

Mode No.	Measured ξ. (%)	Measured *f.* (Hz)	4th FE-update *f.* (Hz) (Block Lanczos)	% Error	% Dev
1	1.4	12	12.63	5.25	2.56
2	2.0	14.7	14.98	1.90	0.94
3	1.2	30	31.22	4.07	1.99
4	1.0	40	43.96	9.90	4.72
5	1.6	67.3	70.95	5.42	2.64
6			99.30		

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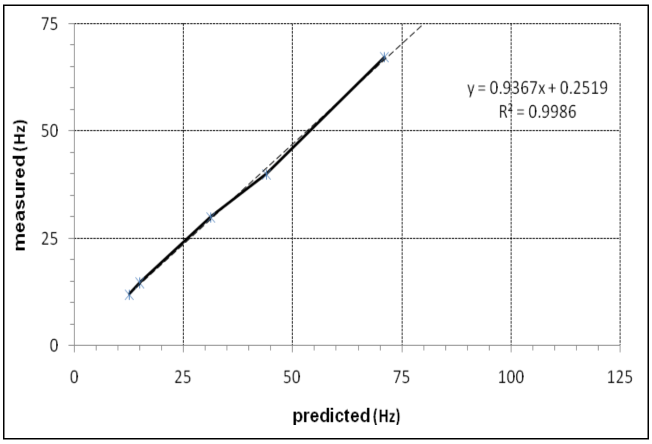


Figure 11. Forth correlation between measured and predicted natural frequencies.

5.2.7 Fine Tuning

Fine tuning was centred mainly on the material properties of concrete and was carried out in an effort to make final improvements to correlation. Hence, the modulus of elasticity of concrete was reduced slightly to 28.5 $kNmm^{-2}$ and its density was increased to 2450 kgm^{-3}. It is accepted that the former is outside the range of values given in Table 4 but within the acceptable values for C35/45 concrete. The density is still within the values given by EC2. The results from this update are presented in Table 12 and the final correlation is plotted in Figure 12. The calculated values of frequency are also listed alongside for comparison.

Table 12. FE Fine-Tuning. Measured, Calculated and FEA–predicted natural frequencies.

	$E_{concrete}$ = 28.5 ($kNmm^{-2}$) $\rho_{concrete}$ = 2450 (kgm^{-3})					
Mode No.	Measured ξ. (%)	Measured *f.* (Hz)	Calculated *f* (Hz)	Fine Tuning *f.* (Hz) RC + Sup-Stiff.	% Error	% Dev
1	1.4	12	11.98	12.12	1.00	0.50
2	2.0	14.7	14.90	14.54	-1.09	-0.55
3	1.2	30	31.80	30.40	1.33	0.66
4	1.0	40	43.85	41.45	3.63	1.78
5	1.6	67.3	70.70	69.80	3.71	1.82
6			112.24	95.7		

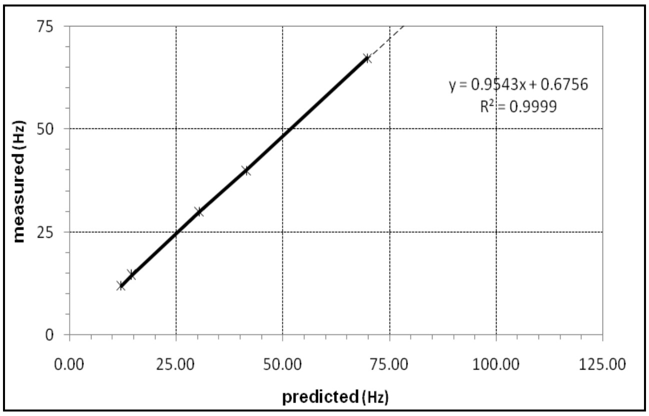
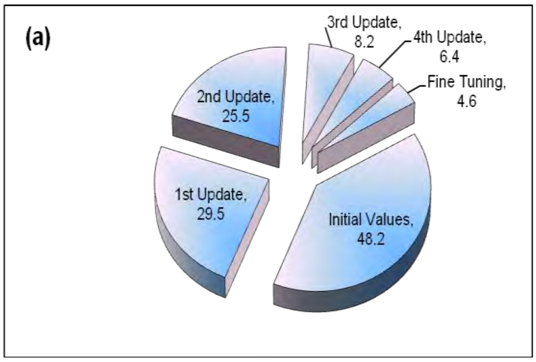


Figure 12. „Fine-Tuning' correlation between measured and predicted natural frequencies.

Fine tuning improved the slope of the „best fit' from 0.936 previously, to 0.954. This is an improvement to target of only 0.018 but it could easily be justified and included in the updating process. The final best-fit value is only 4.6% away from „perfect' correlation.

6. COMMENTARY ON THE RESULTS

The main debate regarding the results has taken place during the updating process in Section 5 but one or two additional comments may be of interest. The final best-fit values were out by 4.6% from the perfect correlation and although efforts were made to improve the above figure this was not easily explained and justified. It is clear from the above that correlation depends on a number of variables but their degree of sensitivity is different. Correlation is particularly sensitive to the physical properties of concrete and the boundary conditions of the structure considered. Figure 13a demonstrates how far the best-fit is from the target value of one, after each updating process. The difference of 18.7% between the initial model and the first update and that of 17.3% between the second and third updates, demonstrate the importance of the material properties and support conditions in the process. A similar conclusion can be drawn from Figure 13b demonstrating relative improvements in the road to perfect correlation.



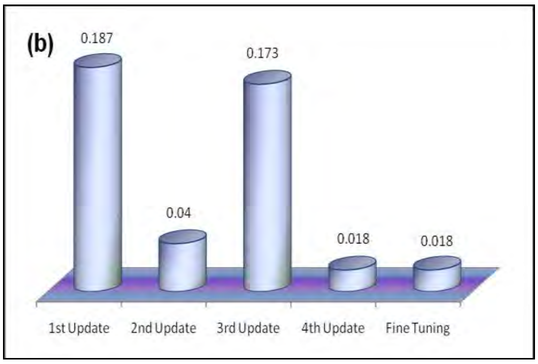


Figure 13 (a). Percentage of „best-fit' out of „perfect' correlation.
(b). Contribution of each updating process to perfect correlation.

Table 13 is dedicated to mode shapes. There is relatively good agreement between mode shapes from the experimental and the finite element modal analysis. Mode 2 is of some interest as an almost predominantly torsional mode. It would be very difficult to interpret it without the help of FEA. Note that plane symmetry has been applied to all FE-models and therefore only half the models are shown. It should be stressed again that without the help of the finite element analysis it would be difficult to interpret any complex modes based on the experimental results alone. Modes 2 and 3 represent twisting tendencies of the L-shaped terrace unit (depicted successfully in the FE analysis) whereas modes 4 and 5, may be regarded as the second and third „bending-like' modes of vibration.

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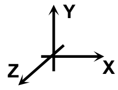
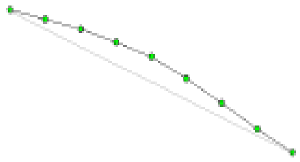
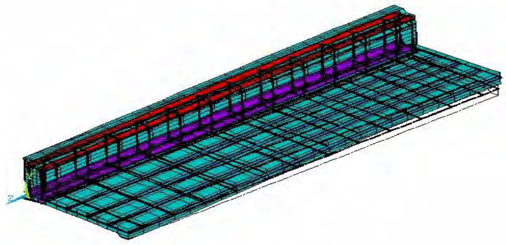
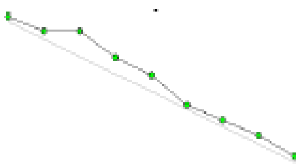
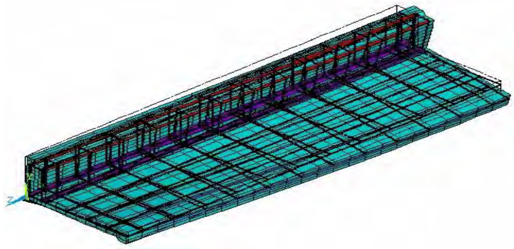
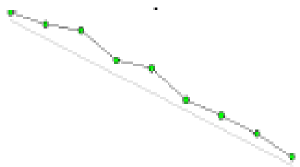
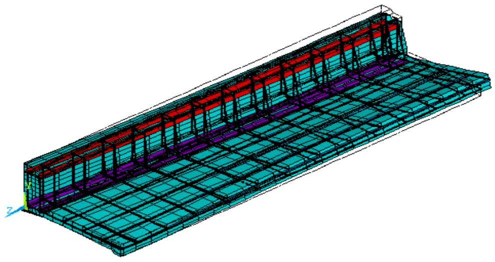
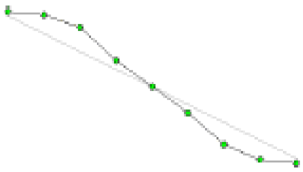
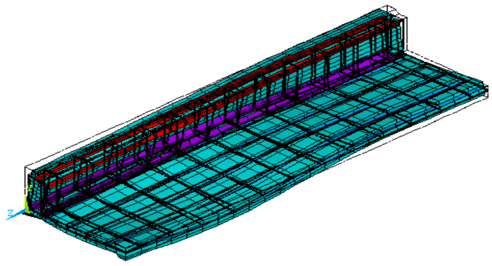
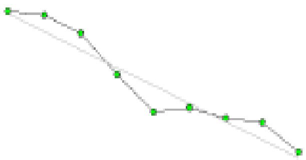
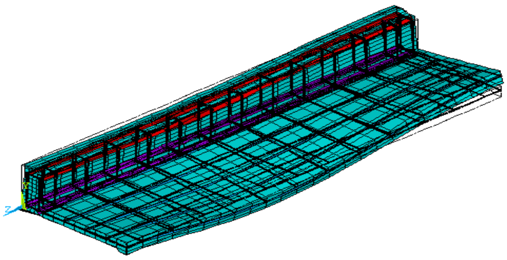
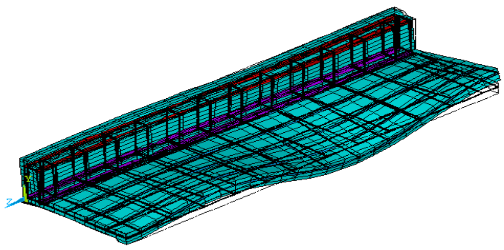
Table 13 Terrace Unit. Mode shapes, estimated experimentally and predicted by FEA.

Mode No.	Experimental Modal Analysis. Mode Shape	FE Modal Analysis. Mode Shape	Comments 
1			The fundamental, bending mode of vibration.
2			Predominantly torsional mode. Also showing small amounts of bending.
3			Similar to Mode 2.
4			The second (flexural) mode of vibration.
5			The third flexural mode Also inhibiting a small amount of torsion.
6	?		Predominantly bending mode

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7. UNCERTAINTIES IN MODAL ANALYSIS

In general, the natural frequencies and mode shapes showed good correlation. As all accelerometers were positioned vertically and the structure under tests was approximated by RPs (lumped masses) in a straight line, it is reasonable to say that the experimental procedure needed assistance from the FEA in order to depict certain complex modes of vibration.

The role the reinforcement plays in the dynamic behaviour of the structure can be of some interest. Numayar *et al* (2003) concluded that steel reinforcement has no effect on the natural frequency of structures provided that the applied loads were kept below that causing the first crack. The same authors concluded that for applied loads greater than the cracking load, the natural frequency increases with reinforcement ratio. Feelings are mixed in this study as no clear pattern emerged. Strictly speaking, when the structure is cracked its stiffness is reduced and so should be its natural frequency. The fact that the latter is increasing means that the loss of stiffness is not significant, which is true (Karadelis 2009a).

Similar studies at MSc level carried out by Pandelli and Karadelis (2003) have demonstrated the change in dynamic behaviour of a simply supported, singly reinforced concrete beam of rectangular section undergoing free vibrations. It was found that increasing the amount of reinforcement is likely to increase certain modal frequencies and decrease others. Subsequently, it was noticed that reinforcing and therefore increasing the „specific' stiffness of the beam may result in "forcing" the beam into a different mode of vibration and lower rather than raise its corresponding frequency.

It is apparent that the geometric properties of the units (asymmetric cross-section) have an effect on its flexural and torsional rigidities (EI & GJ). Clearly, the stiffness of the unit along its main span is different to that along the short span. Furthermore, as the section along the short span varies, so does its stiffness and therefore a third stiffness value may be justifiable. Stiffness reduction related to possible cracks (shrinkage) may also be a possibility. Efforts were concentrated to assess the different stiffnesses of the unit and represent them in a smeared manner in the updating process. As a first approximation, it was decided to calculate the stiffness of the different parts of the unit such as the tread, the riser, etc and then assign modified E-values to these parts. However, this approach could not be justified due to its extremely long computational effort and is not shown here. A purely analytical modal analysis is currently under development by the author where uncertainties like the above can be portrayed more accurately.

The verification process does not, normally, consist part of the modal analysis procedure but some clarifications are of interest. The procedure involves the development of three partial differential equations from where the "exact" solution may be extracted. Inevitably, the equations developed depend on a series of constants such as: E_{con}, G_{con}, $I_{Y\text{-}Y}$, $I_{X\text{-}X}$, I_{xy}, I_{sc}, J, I_w, ρ, e_x and e_y, as well as their products and sums (EI_{sc}, EI_w, EI_{xy}, GJ, $I_x.I_y.I_{sc}$, $(I_x + I_y)$, $(I_xI_y - I^2_{xy})$) that can only be evaluated approximately. In addition, as most terms of the final frequency equation are raised to some high power the initial error is exaggerated and accuracy is compromised.

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8. CONCLUDING COMMENTS

A comprehensive account of experimental and numerical modal analysis for a family of reinforced concrete grandstand terraces, supported on a rectangular hollow section (RHS) frame was presented in this study. Based on the findings, the following conclusions can be drawn:

1. Overall, and after studying the first six modes of vibration it was found that experimentally obtained natural frequencies were in good agreement with the ones predicted by a continually updated finite element model.

2. Experimental procedures may not be adequate to provide a complete account of modal analysis unless the equipment used is in abundance, of good quality, high resolution and standards and therefore extremely expensive and unsustainable for research work alone. Hence, some limitations are inevitable as certain modes that tend to be more complex than others, may not be depicted accurately by the experimental analysis alone. A rigorous finite element model should help to overcome this problem by capturing the resulting mode shapes with confidence and accuracy, especially at higher modes of vibration.

3. Initial and short term results suggest that the amount of reinforcement has little effect on the dynamic properties of the "uncracked" reinforced concrete terrace units. However, interim but carefully studied results from the finite element analysis hinted towards the possibility that an increase in the amount of reinforcement is likely to force the structure into a different mode of vibration, hence altering the previously obtained properties.

4. The dynamic properties of the terrace units were found to be very sensitive to two parameters, the physical properties of the main material (concrete) and the conditions of the supports of the structure, as expected. The degree of sensitivity is discussed in some detail in Sections 5 and 6. Essentially, it was found that a gradual improvement of the predicted natural frequency values was evident by progressively improving the representation of boundary conditions. Best correlation was achieved when the inserted between the units neoprene pads were modelled by simulating their stiffness characteristics with the ANSYS dedicated stiffness matrix element.

5. Also, it was found (section 5.1) that the subsequent adoption of Block Lanczos solution technique, over the initial choice of the subspace method, reduced the CPU-time significantly and made a relatively modest contribution to the accuracy of the results. It is therefore recommend for similar type of numerical analysis work.

6. Verification of the experimental and numerical results may not always be possible, or may sometimes be impractical due to complex mathematical calculations and a plethora of uncertainty factors arising mainly from the constants involved.

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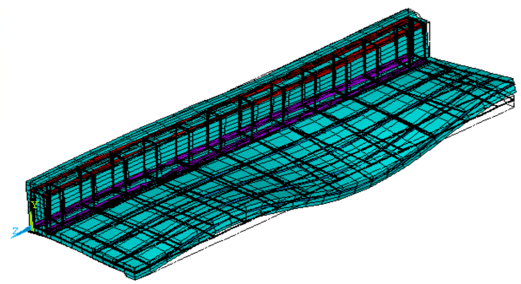
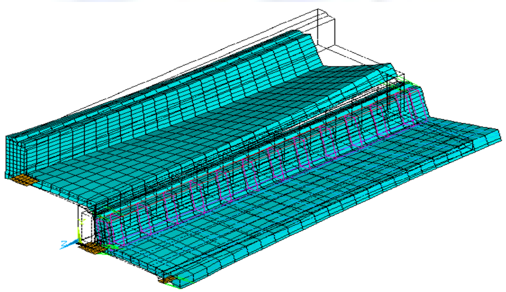
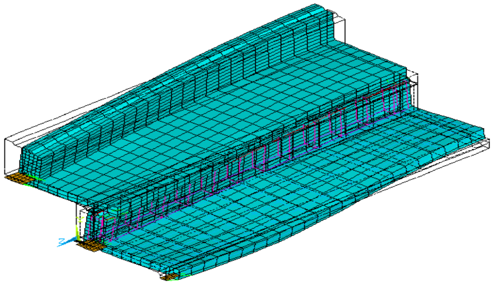
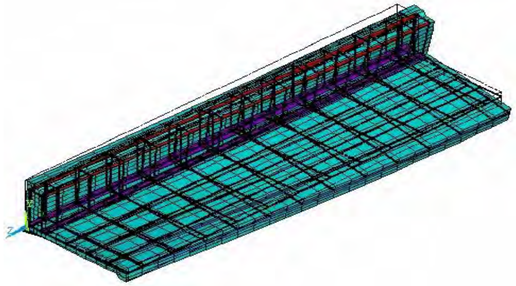








John N Karadelis

Structuring

1. Intro.
   Contribution & Objectives

2. Areas of Interest
   Concrete Pavements & Grandstand Vibrations

3. Articles Submitted
   Object, Hypothesis, Contributions, Benefits, Originality & Impact.

4. Closing Remarks
   What the research has offered.

5. Outlook
   Limitations. Drawbacks. More research.

Coventry University

1. INTRODUCTION

❖ **Main Contribution?**
The successful solution of complex problems in Civil and Structural Engineering by means of advanced and innovative computer simulations and numerical techniques.

❖ **How?**
By successfully modelling the behaviour of structures, materials, complex geometries, loadings, boundary conditions, etc.

❖ **Why Reinforced Concrete?**
Because it constitutes a relatively untouched and complex area in computational mechanics and connoted the essential niche.

Main Objectives

- ✓ **Identify** the general convenience, accuracy, savings and quality, associated with computational modelling in engineering.
- ✓ **Develop** series of explicit computer models based on latest advances in numerical methods in engineering and reinforced concrete technology.
- ✓ **Use** models to gain proficiency and predict the real behaviour of certain structures.
- ✓ **Extend** methodologies to other engineering structures.
- ✓ **Highlight** advantages, drawbacks, limitations, methods, techniques, procedures and solutions employed.
- ✓ **Diminish** (gradually), or … replace costly exp'l work.

2. Areas of Interest

Sustainable Concrete Pavement Overlays

Aim:

To show that bonded and suitably reinforced, polymer enriched, concrete overlays, constructed by utilising asphaltic paving techniques and equipment, is a viable, sustainable, environmentally friendly, option.

Unique Qualities:

* Maximum use of existing investment (taking advantage of residual strength of old pavement).
* Combine low cost and long term sustainable solutions with concern for the environment (avoiding wholesale replacement of damaged pavement).

Grandstand Vibrations due to Human Movements

Four highly recommended research areas were identified

(JWG of IStructE)

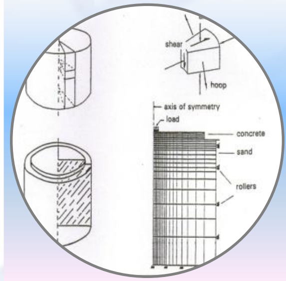
JWG IStructE

- Full scale Experimentation & Long-term Monitoring of relevant structures.
- Derivation of Load types and Loading Scenarios generated by lively crowds.
- Human Structure Interaction (unknown and complex area).

JNK

- More Accurate Computer Simulation of Structures.

3. The Submitted Articles



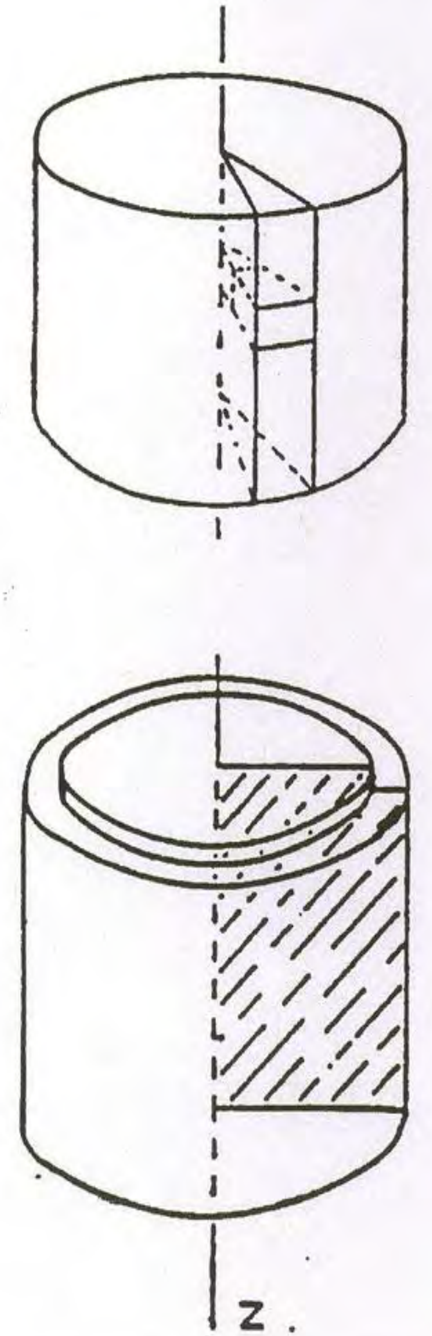
Article I. 'A Numerical Model for the Computation of Concrete Pavement Moduli: A Non-destructive Testing and Assessment Method'. *NDT&E International*, 2000.

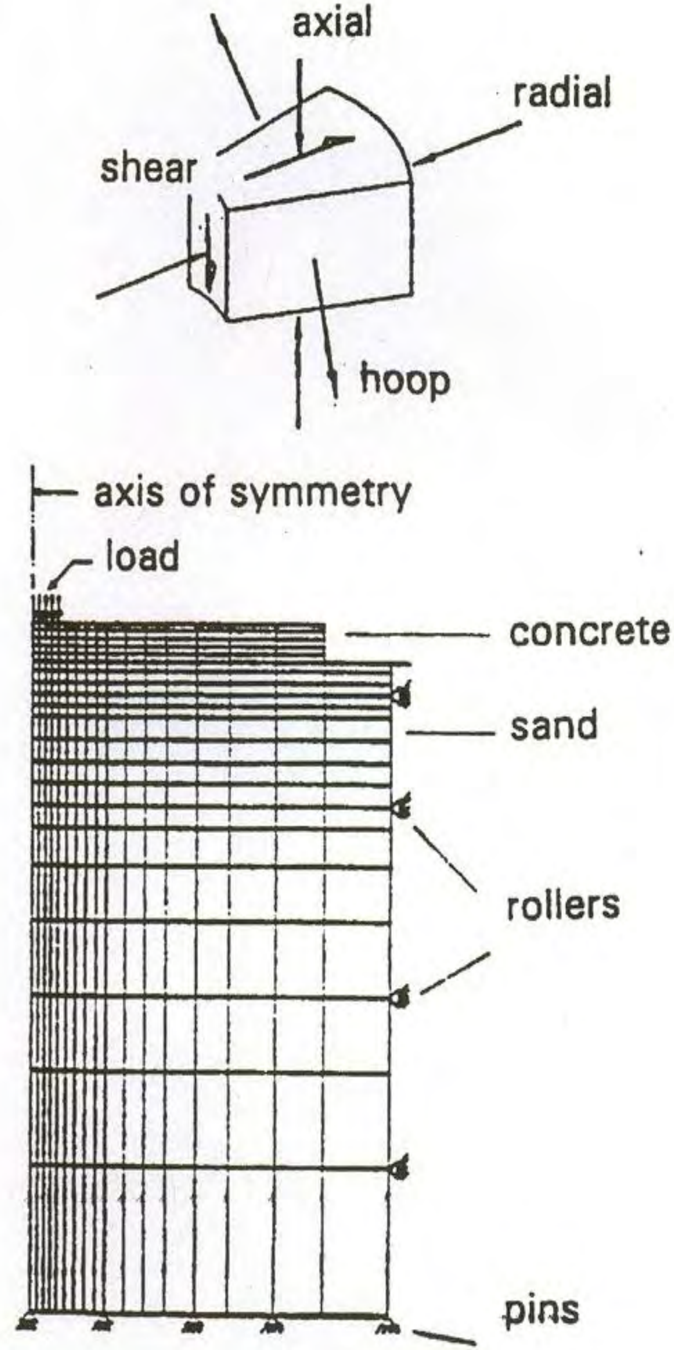
object:
Develop a NDT method capable of assessing the structural condition (evaluation) of a rigid, layered pavement by utilising the FWD machine. (funded by BAA & Gatwick Airport)

hypothesis:
If surface deflections, stresses and strains within a pavement were measured simultaneously and under given load, and then correlated with theoretical values obtained from a similar, purposely calibrated FE-model, then the assumed theoretical moduli should be representative of the pavement.

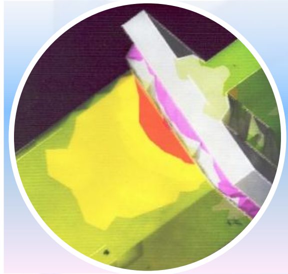
Contributions, Benefits, Originality, Impact :

- ✓ Development of a **NDT** method _The „rigid' **FWD.**
- ✓ **Cubic Spline, Exponential function. _** math represent'n
- ✓ **Shear Transfer** problem resolved. Introducing concept of **Mechanical Efficiency** in algorithm.
- ✓ Disclosure of **findings**, **advantages** and **shortcomings** of FWD, to engineering consultants and contractors.
- ✓ „**Know-How**' to other researchers. _Publ's and Citations.
- ✓ Demonstrating **Novelty**, **Flexibility**, **Effectiveness.**
- ✓ Large No. of **Citations** _show significant impact.





Cylindrical solid subjected to axi-symmetric load and corresponding FE-model, featuring 2D iso-parametric elements



Article II. 'Elasto-Plastic FE-Analysis with Large Deformation effects of a T-end Plate Connection to Square Hollow Section' *Finite Elements in Analysis and Design*, 2001.

object:

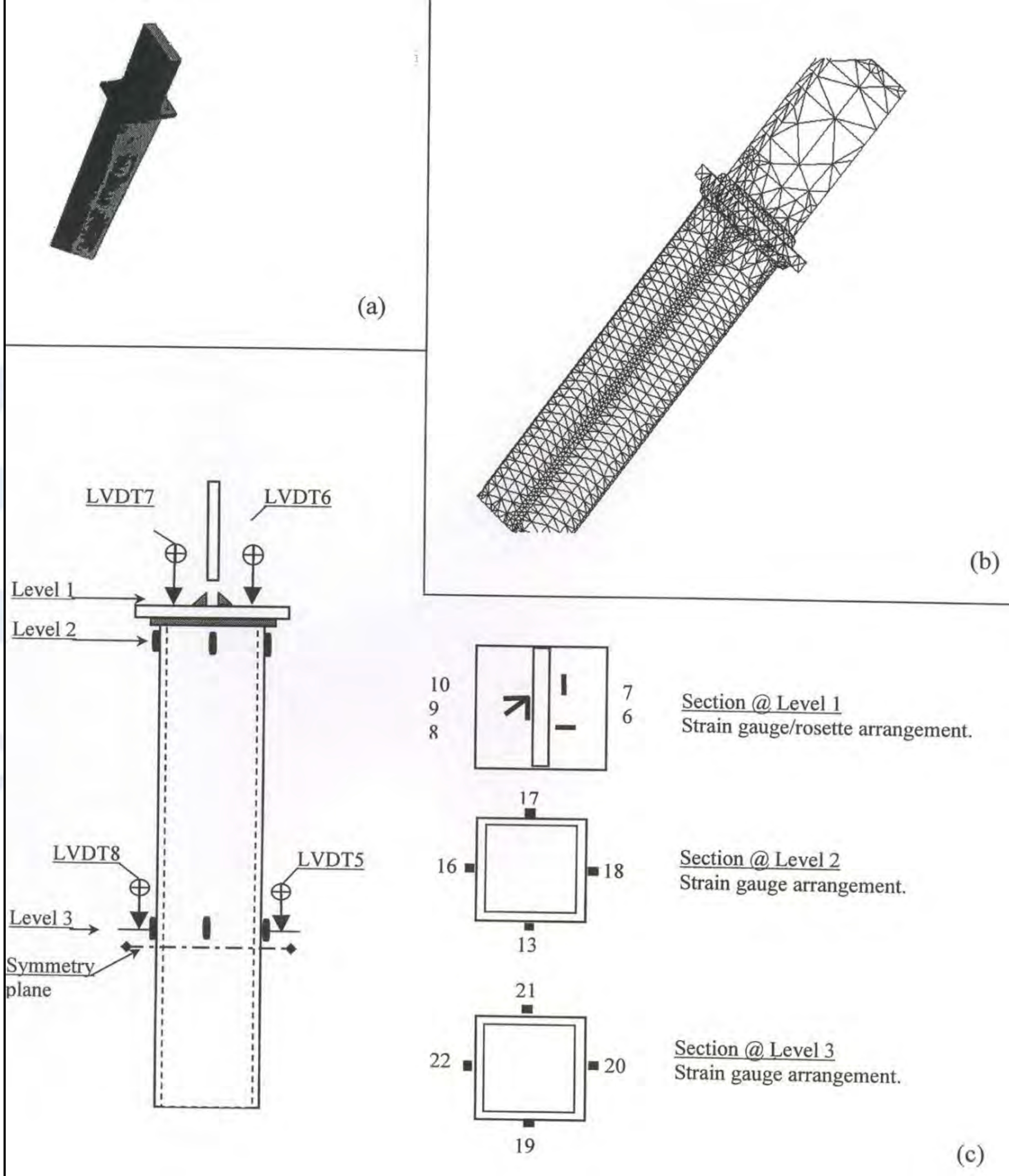
The performance of a popular type of structural steel SHS connection. Aimed to investigate and optimise connection by integrating two distinct types of analyses (later).

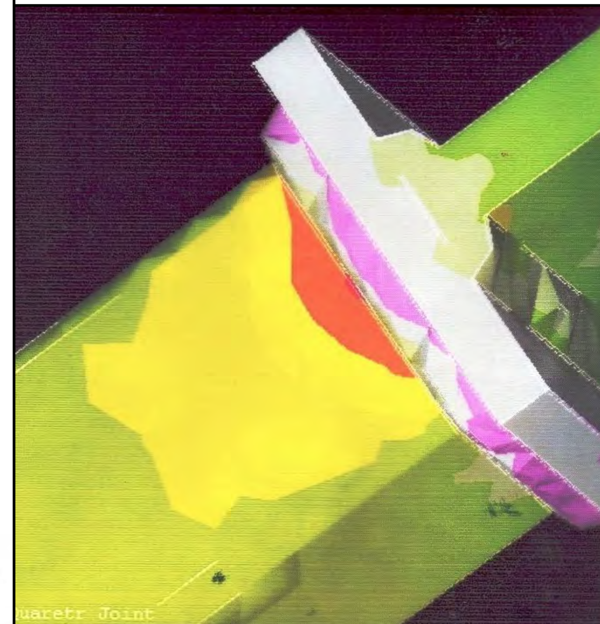
hypothesis:

- ✓ If a parametric study in non-linear domain (large deflections) was carried out, and the sensitivity of parameters affecting cap and cleat plates was identified,
- ✓ Then, study could be use to optimise thickness of fin plate and advise for optimum design.

Contributions, Benefits, Originality, Impact:

- ✓ **Added** to existing knowledge by introducing material (bi-linear) and geometric (LD) non-linearities in numerical analysis. _Very successful representation for steel
- ✓ Guidelines for the **optimum design** of a family of structural connections.
- ✓ „**Alerted**' others of large deformation effects and their integration in analysis procedure.
- ✓ Optimising performance of each individual component **separately**, better than considering connection as a whole.
- ✓ Number of **citations** reinforces usefulness of model.





Line diagram of a SHS showing position of transducers.

FE model of the same assembly featuring 3D tetrahedral elements

Article III. 'Concrete Grandstands. Part I: 'Experimental Investigation', *Proceedings of the Institution of Civil Engineers: Engineering and Computational Mechanics*, 2009.

object:
Comprehensive experimental investigation of a set of precast concrete terrace units. Comment on their behaviour under static loading-unloading. Estimate the uncracked and cracked stiffness of the units. Use them to calibrate and fine-tune FE-models.

hypothesis:
If the uncracked and cracked stiffness of the units were estimated experimentally and used as initial input in a dynamic analysis, then the adopted FE-models should be justifiable, accurate and representative.

Contributions, Benefits, Originality, Impact:

- ✓ **Novel** experimental investigation;
- ✓ Good **quality data** for designers to revise their design philosophies. _(SS-design idea?)
- ✓ **Stiffness** values for uncracked and cracked sections for further FE-modelling work.
- ✓ Established **theories** (ε-dist'n across section, 'reduction' in stiffness) also apply here.
- ✓ No universally accepted constitutive law concrete exists. Hence, experimental "**add-ons**" always welcome.
- ✓ Findings can be used by other researchers when validating own models. _**Stepping Stone?**

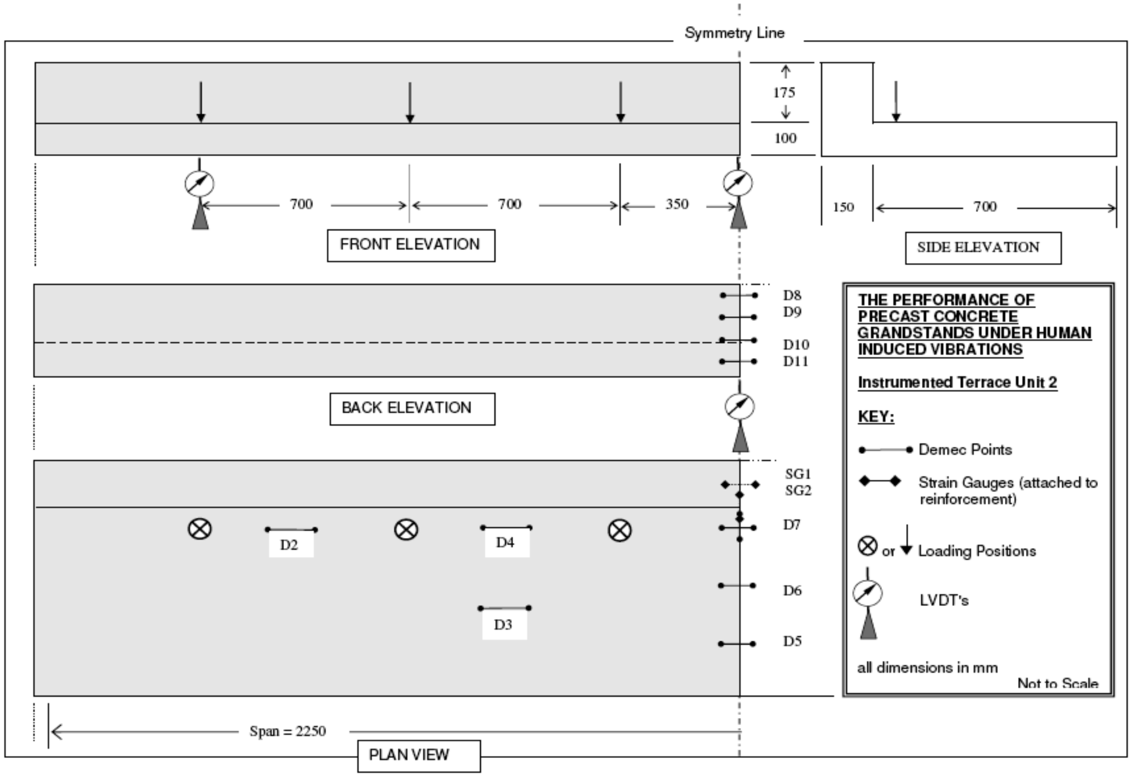
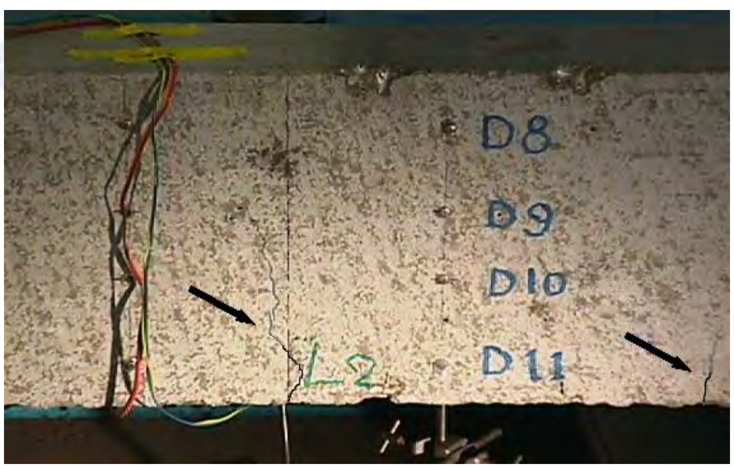
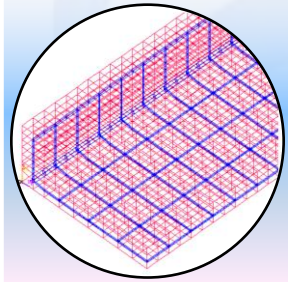


Diagram of the Instrumentation of the unit.

Terrace unit in the laboratory and …



Demec Points capturing the first crack.



Article IV. 'Concrete Grandstands. Part II: Numerical Modelling', *Proceedings of the Institution of Civil Engineers: Engineering and Computational Mechanics*, 2009.

object:

Portrays the development of a rigorous FE model with distinct material non-linearities for both, concrete and steel reinforcement. Failure criteria for materials were based on their discrete modes and mechanisms of failure.

hypothesis:

Accurate RC numerical models not in abundance to date.

If realistic failure criteria (cracking-crushing, yielding), were integrated in analysis, then the resulting models should be representative and the contribution to the subject significant.

Contributions, Benefits, Originality, Impact:

- ✓ General elasto-plastic, constitutive, **meso-macro scale level model** of a RC structure, featuring **cracking and crushing** options for concrete failure and **yielding** for steel. Useful to practicing engineers designing beyond conventional codes of practice.

- ✓ Demonstrated and cautioned the engineering community that **Crisfield's arc-length** solver, does not always produce accurate results.

- ✓ There is no such an algorithm to-date capable of capturing successfully the **descending part** of a stress-strain curve of certain materials.

- ✓ More realistic, **brittle failure** of concrete, as opposed to strain softening, discussed and reasoned.

Contributions … Impact (continued)

- ✓ **Contribution** of main reinforcement to shear resistance addressed by attributing it to surrounding aggregate interlocking ability.

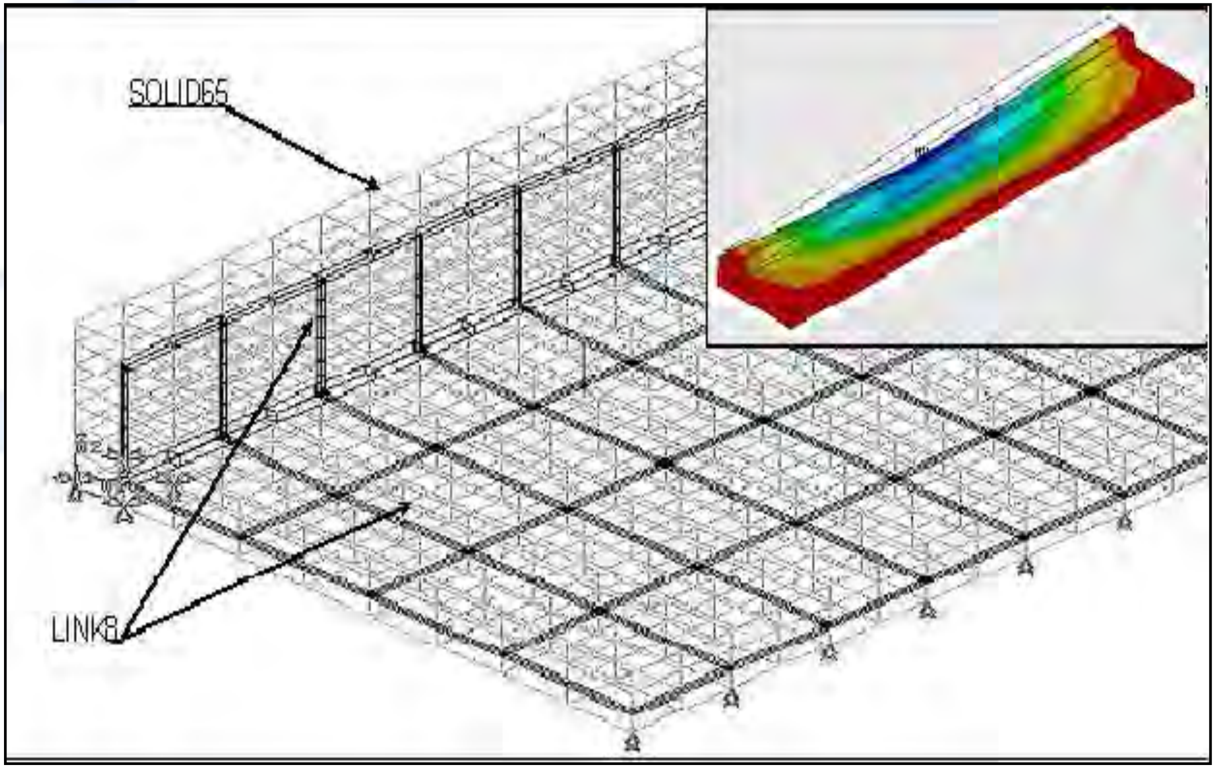
- ✓ Addresses problem of **shear transfer** in a simple and effective manner (lack of suitable element).

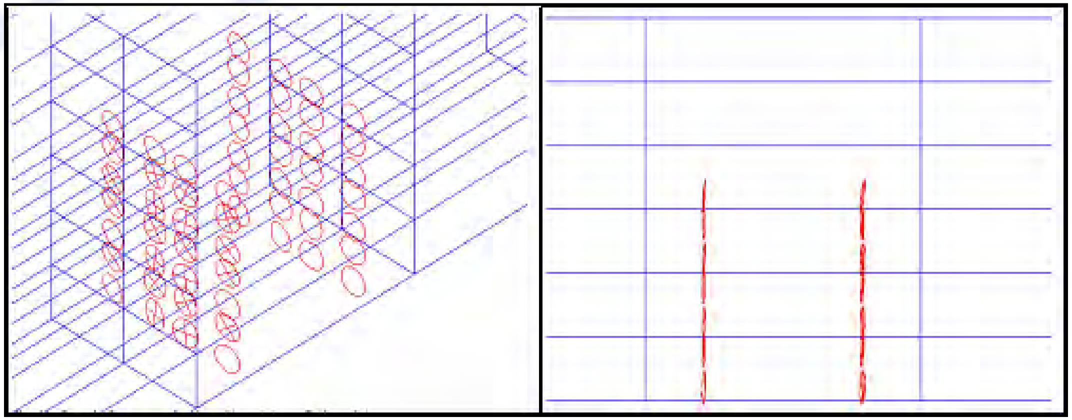
- ✓ **Capturing** of flexural cracks has opened new prospects in modelling failure conditions of RC structures.

- ✓ Macro/meso scale simulation provided reliable results with reasonable effort. Recommended for modelling **large structures.**

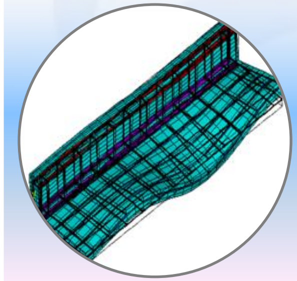
Finite Element model of a terrace unit featuring SOLID65 and LINK8 elements.

Inset: Predicted vertical displacement contours





Flexural crack prediction, similar to that shown earlier, in Part I, Exp. Investigation.



Article V. *'Grandstand Terraces. Experimental and Computational Modal Analysis.' 8th World Congress on Computational Mechs, & 5th EU Congress on Computational Methods in Applied Sciences, ECCOMAS 2008*:

object:
„Transformation' of static grandstand terrace models into dynamic, as well as the experimental and numerical estimation of their corresponding properties. Suggest ways to adjust and fine-tune the models and attempt a first comparison between numerical and experimental results.

hypothesis:
A successful FE model developed earlier in statics, can provide a firm, initial platform in dynamics, integrating the two domains. Modifications and fine tuning could take place relatively easily afterwards.

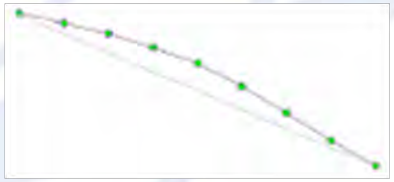
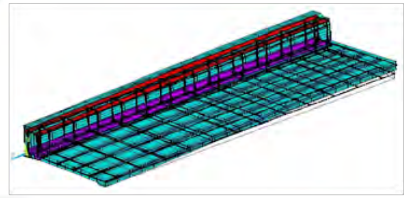
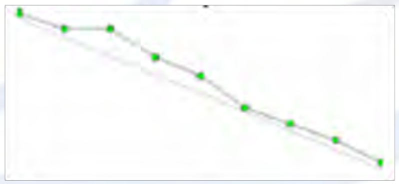
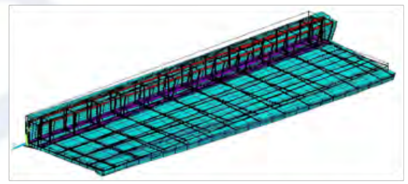
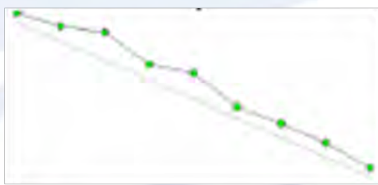
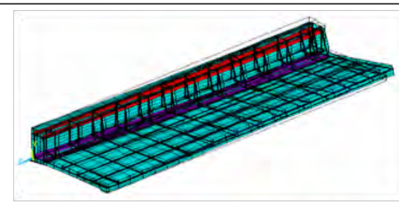
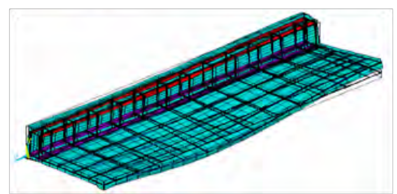
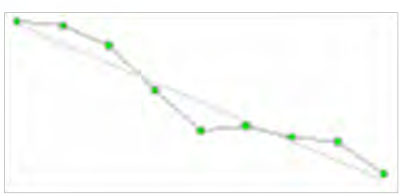
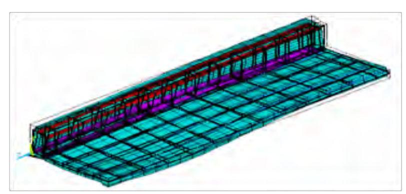
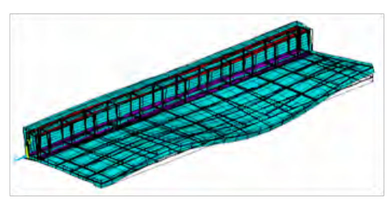
Contributions, Benefits, Originality, Impact:

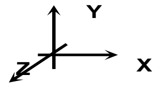
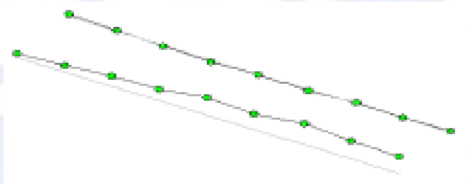
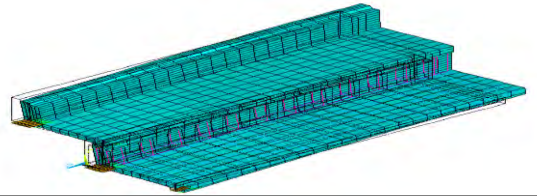
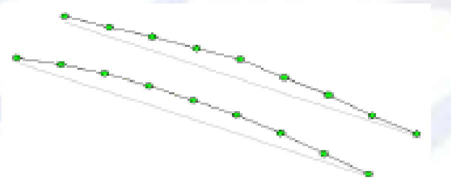
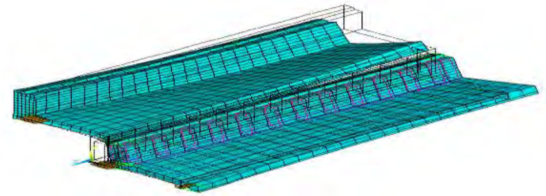
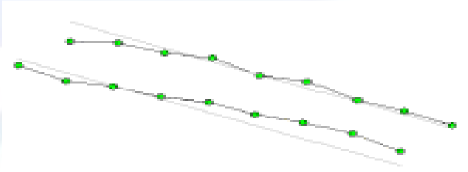
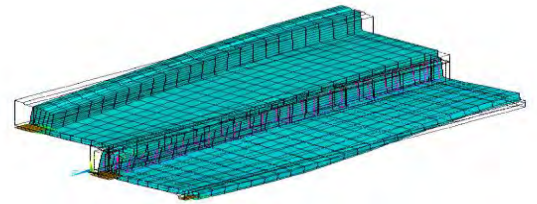
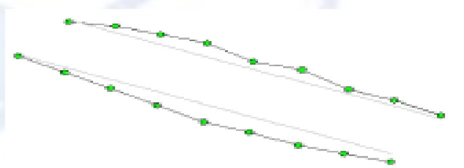
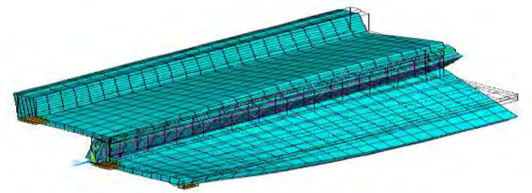
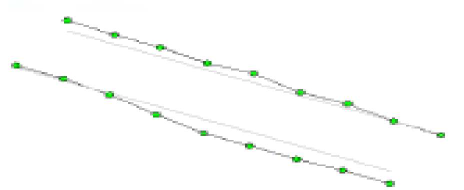
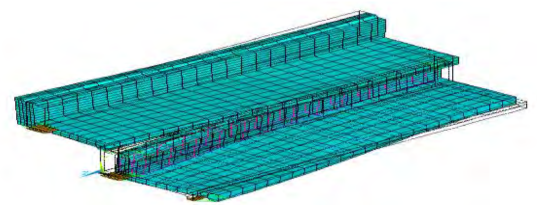
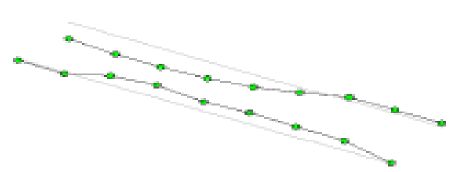
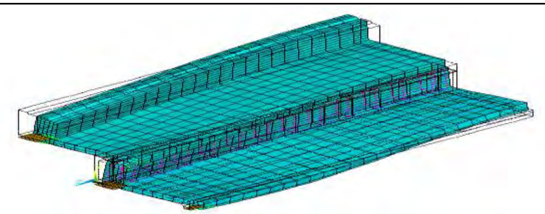
- ✓ **Experimental** modal analysis should be augmented by **FEA**. Better understanding of the dynamic properties of a structure.

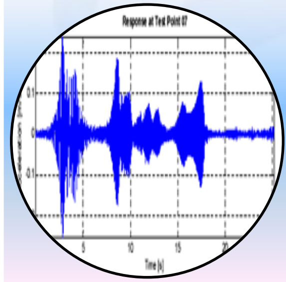
- ✓ **Reinforcement** can change dynamic performance of a structure by reverting from one mode of vibration to another.

- ✓ Studying **parts** of a large structure alone, may not provide reliable conclusions regarding the dynamic performance of the latter (revisited later).

- ✓ Indications that, ***sub-structuring / super-element*** techniques can provide better global understanding (on-going research objective).

Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	*f* (Hz) Ex p. Theo . ζ_{exp} (%)	Comments
1			12. 1 0 12. 24 1.4	The fundamental bending mode of vibration.
2			14.7 0 1 4.55 2.0	Predominantly torsional. May also be showing small amount of bending.
3			30.0 6 30 .7 0 1.2	Similar to Mode 2.
4			40. 0 1 4 4 .56 1.0	The second (flexural) mode of vibration.
5			67.3 7 1 . 4 1.6	Near perfect flexural mode with a small amount of torsion.
6	?		103.00	Bending Mode

Mode No.	Experimental Modal Analysis Mode Shape	FE Modal Analysis Mode Shape	*f* (Hz) Exp. Theo. ζ_{exp} (%)	Comments 
1			9.30 9.40 2.2	Upper unit is static. Lower unit lifts corner over support (Rigid Body)
2			11.20 11.72 1.5	First bending & slightly twisting mode. "In phase" vibration
3			16.00 14.08 2.5	Coupled. Bending & twisting in unison about Z-axis. "In-phase" motion.
4			17.00 17.06 2.0	Mainly twisting mode about Z-axis "Out-of-phase" Vibration.
5			22.4 29.40 2.6	Upper unit: "Rigid Body" twisting about Y-axis. Lower unit: Mainly bending.
6			29.1 30.80 3.2	A complex, mainly torsional, and flexural vibration.



[Article VI.]() 'Reliability Pointers for Modal Parameter Identification of Precast Concrete Terraces'. *Computers and Concrete An International Journal*, 2010 (accepted, currently under revision).

object:

Demonstrates efforts to continue with research on grandstand vibrations. Focuses on the numerical side of modal analysis, and how a good numerical research strategy can capture effectively the modal parameters of a structure.

hypothesis:

By developing and importing successful RC numerical models from statics, one can obtain accurate structural information in dynamics, by identifying a series of „reliability pointers' through appropriate, and quantifiable and justifiable structural „tuning'.

Contributions, Benefits, Originality, Impact:

- ✓ **Boundary conditions** important in dynamic performance of structures.
- ✓ **Initial** properties input, as well as **changes** in properties (stiffness) during analysis process, are imperative.
- ✓ Use of **FEM** means savings, accuracy and efficiency.
- ✓ “**Pointers**” & “**Index of accuracy**” introduced, can justify some extra work in the updating process.
- ✓ Degree of accuracy decides if „**insensitive**’ parameters may be left out (eg: distribution steel).
- ✓ Full **justification** of updated parameters is possible using the proposed method.

4. Closing Remarks

- More articles by author. _In Int'l journals, proceedings, conferences, workshops, lectures and presentations (CV).
- Has tried to **disseminate** research knowledge to students and other fellow academics by **blending** it in his lectures and **publishing** in engineering and pedagogical journals.
- Hopes he has earned credible reputation as a research scholar specialising in **Applied Computational Mechanics** and has reserved a place in academic and engineering practice communities.

5. Outlook

- Likely that author has presented a **critical analysis** based on his own material and other sources and has developed some **original approaches** to **numerical modelling** in engineering.

- Well aware of the **limitations** of his own research and **margins** his numerical models operate.
  He knows that he has not, as yet, addressed fully the **implications** of these limitations and their **impact** on numerical simulation.

  - For instance: Failure mechanism of bond (**debonding**), between re-bars and the surrounding concrete at meso/macro scale level modelling, still to be addressed.

  - Also: Computer representation of **complex loads** signifying human activities remains unanswered.

Consequently, author’s future research plans include further and more focused research into **on-going** but also **new** areas:

Green Overlays. Through his research (four) students and the relevant EPSRC grant he currently holds.

Grandstand Vibrations. Continue to work and experiment with large scale structures and their accurate computer representation.

New, ideas and areas of applied research:

- Noise and vibration generated from building mounted micro-**wind turbines** (new research student – half scholarship).
- **Punching shear failure** of flat slabs supported by steel columns (research student – funded by own government

Ultimately,

Author hopes has accomplished a **sustained and systematic improvement over time**, resulting in his own development and contribution to knowledge and practice.

Thank you

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On that b... onsidered for the higher degree o... b... Articles.

Coventry University